CHAPTER 6

Flow of Fluids

Fluids are substances that flow without disintegration when pressure is applied. This definition of a fluid includes gases, liquids, and certain solids. A number of foods are fluids. In addition, gases such as compressed air and steam are also used in food processing and they exhibit resistance to flow just like liquids. In this chapter, the subject of fluid flow will be discussed from two standpoints: the resistance to flow and its implications in the design of a fluid handling system, and evaluation of rheological properties of fluid foods.

6.1 THE CONCEPT OF VISCOSITY

Viscosity is a measure of resistance to flow of a fluid. Although molecules of a fluid are in constant random motion, the net velocity in a particular direction is zero unless some force is applied to cause the fluid to flow. The magnitude of the force needed to induce flow at a certain velocity is related to the viscosity of a fluid. Flow occurs when fluid molecules slip past one another in a particular direction on any given plane. Thus, there must be a difference in velocity, *a velocity gradient*, between adjacent molecules. In any particular plane parallel to the direction of flow, molecules above and below that plane exert a resistance to the force that propels one molecule to move faster than the other. This resistance of a material to flow or deformation is known as stress. *Shear stress* (τ) is the term given to the stress induced when molecules slip past one another along a defined plane. The velocity gradient ($-dV/dr$ or γ) is a measure of how rapidly one molecule is slipping past another, therefore, it is also referred to as the *rate of shear*. The position from which distance is measured in determining the shear rate is the point in the flow stream where velocity is maximum, therefore, as distance r increases from this point of reference, V decreases and the velocity gradient is a negative quantity. Because shear stress is always positive, expressing the shear rate as $-dV/dr$ satisfies the equality in the equation of shear stress as a function of shear rate. A plot of shear stress against shear rate for various fluids is shown in Fig. 6.1.

Fluids that exhibit a linear increase in the shear stress with the rate of shear (Eq. 6.1) are called *Newtonian fluids*. The proportionality constant (μ) is called the *viscosity*.

$$\tau = \mu\left(-\frac{dV}{dr}\right) \tag{6.1}$$

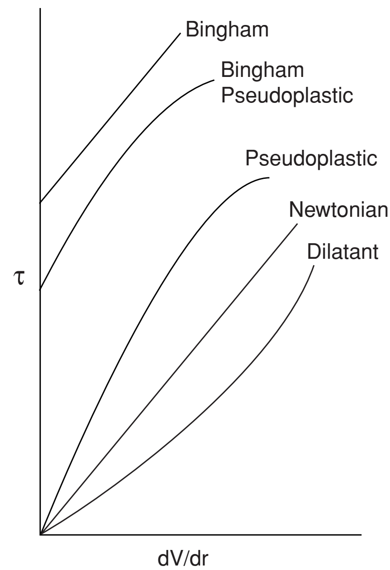

Figure 6.1 A plot showing the relationship between shear stress and shear rate for different types of fluids.

Newtonian fluids are those that exhibit a linear relationship between the shear stress and the rate of shear. The slope μ is constant, therefore the viscosity of a Newtonian fluid is independent of the rate of shear. The term viscosity is appropriate to use only for Newtonian fluids. Fluids with characteristics deviating from Equation (6.1) are called *non-Newtonian* fluids. These fluids exhibit either shear thinning or shear thickening behavior, and some exhibit a yield stress (i.e., a threshold stress that must be overcome before the fluid starts to flow). The two most commonly used equations for characterizing non-Newtonian fluids are the power law model (Eq. 6.2) and the Herschel-Bulkley model for fluids (Eq. 6.3):

$$\tau = K(\gamma)^n \tag{6.2}$$

$$\tau = \tau_o + K(\gamma)^n \tag{6.3}$$

Equation (6.2) can fit the shear stress versus shear rate relationships of a wide variety of foods. Equation (6.2) can also be used for fluids which exhibit a yield stress as represented by Equation (6.3), if τ_o is very small compared to τ, or if the shear rate is very high. n is the *flow behavior index* and is dimensionless. K is the *consistency index* and will have units of pressure multiplied by time raised to the *n*th power.

Equations (6.2) and (6.3) can be rearranged as follows:

$$\tau = [K(\gamma)^{n-1}]\gamma \tag{6.4}$$

$$\tau - \tau_o = [K(\gamma)^{n-1}]\gamma \tag{6.5}$$

Equations (6.4) and (6.5) are similar to Equation (6.1), and the factor that is the multiplier of γ is the apparent viscosity defined as follows:

$$\mu_{app} = K(\gamma)^{n-1} \tag{6.6}$$

For fluids having characteristics which fit Equations. 6.4 and 6.5, the apparent viscosity can also be expressed as:

$$\mu_{app} = \tau/\gamma \tag{6.7}$$

$$\mu_{app} = (\tau - \tau_o)/\gamma \tag{6.8}$$

Substitution of Equation (6.2) for τ in Equation (6.7) also yields Equation (6.6).

The apparent viscosity has the same units as viscosity but the value varies with the rate of shear. Thus, when reporting an apparent viscosity, the shear rate under which it was determined must also be specified.

Equation (6.6) shows that the apparent viscosity will be decreasing with increasing shear rate if the flow behavior index is less than 1. This type of fluids exhibits "shear thinning" behavior and are referred to as *shear thinning fluids* or *pseudoplastic fluids*.

When the flow behavior index of a fluid is greater than 1, μ_{app} increases with increasing rates of shear and the fluid is referred to as *shear thickening* or *dilatant* fluids. Shear thinning behavior is exhibited by emulsions and suspensions where the dispersed phase has a tendency to aggregate following the path of least resistance in the flow stream, or the particles align themselves in a position that presents the least resistance to flow. Shear thickening behavior will be exhibited when the dispersed phase swell or change shape when subjected to a shearing action, or when the molecules are so long that they tend to cross-link with each other, trapping molecules of the dispersion medium. Shear thickening behavior is seldom observed in foods, although it has been reported in suspensions of clay and high molecular weight organic polymers.

Fluids that exhibit a yield stress and a flow behavior index of 1, are referred to as *Bingham plastics*. Some foods exhibit a yield stress and a flow behavior index less than 1, and there is no general term used for these fluids, although the terms *Herschel-Bulkley* and *Bingham pseudoplastic* have been used.

6.2 RHEOLOGY

Rheology is the science of flow and deformation. When a material is stressed, it deforms and the rate and nature of the deformation that occurs characterizes its rheological properties. The science of rheology can be used on solids or fluids. Some food materials exhibit both fluid and solid behavior and these materials are called *viscoelastic*. In general, *flow* indicates the existence of a velocity gradient within the material, and this characteristic is exhibited only by fluids. The subject of rheology in this section covers only flow properties of fluids. In foods, rheology is useful in defining a set of parameters that can be used to correlate with a quality attribute. These parameters can also be used to predict how the fluid will behave in a process and in determining energy requirements for transporting the fluid from one point in a processing plant to another.

6.2.1 Viscometry

Instruments used for measuring flow properties of fluids are called viscometers. Newtonian viscosity can be easily measured since only one shear rate needs to be used and therefore viscometers for this purpose are relatively simple compared to those for evaluating non-Newtonian fluids. Viscometers require a mechanism for inducing flow that should be measurable, a mechanism for measuring the applied force, and the geometry of the system in which flow is occurring must be simple in design such that the force and the flow can be translated easily into a shear stress and shear rate.

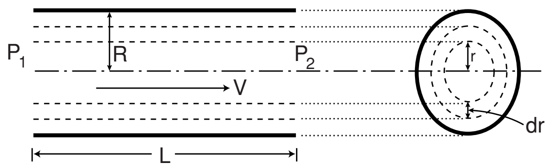

Figure 6.2 Differential control element for analysis of fluid flow through a tube.

6.2.1.1 Viscometers Based on Fluid Flow Through a Cylinder

These viscometers are called capillary or tube viscometers, depending on the inside diameter. The principle of operation is based on the Poiseuille equation, if the fluid is Newtonian. The Rabinowitsch-Mooney equation applies when the fluid is non-Newtonian. The Poiseuille equation will be derived based on the Newtonian flow equation (Eq. 6.1), and the Rabinowitsch-Mooney equation will be derived based on the power law equation (Eq. 6.2).

6.2.1.2 Derivation of the Poiseuille Equation

Figure 6.2 shows a tube of length L and radius R. A pressure P_1 exists at the entrance and P_2 exists at the end of the tube. At a distance r from the center, a ring of thickness dr is isolated, and at this point, the fluid velocity is V, the point velocity. The shear stress at this point is force resisting flow/area of the fluid undergoing shear. The force resisting flow on the fluid occupying the area of the ring is the pressure drop times area of the ring, and the area of the plane where the fluid is sheared is the circumferential area of the cylinder formed when the ring is projected through length, L.

$$\tau = \frac{(P_1 - P_2)(\pi r^2)}{2\pi r L} = \frac{(P_1 - P_2)\, r}{2L} \tag{6.9}$$

Substituting Equation (6.9) in Equation (6.1):

$$\frac{(P_1 - P_2)\, r}{2L} = \mu\left(-\frac{dV}{dr}\right)$$

Separating variables and integrating:

$$\int dV = \frac{(P_1 - P_2)}{2L\mu}\int -r\,dr$$

$$V = \frac{(P_1 - P_2)}{2L\mu}\left(\frac{-r^2}{2}\right) + C$$

The constant of integration can be determined by applying the boundary condition: at $r = R$, $V = 0$. $C = (P_1 - P_2)(R^2)/4L\mu$.

$$V = \frac{(P_1 - P_2)}{4L\mu}(R^2 - r^2) = \frac{\Delta P}{4L\mu}(R^2 - r^2) \tag{6.10}$$

Equation (6.10) gives the point velocity in a flow stream for any fluid flowing within a cylindrical tube. The point velocity is not very easily measured. However, an average velocity defined as the volumetric flow rate/area can be easily measured. Equation (6.10) will be expressed in terms of the

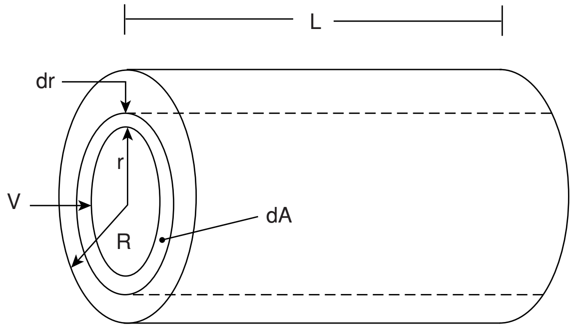

Figure 6.3 Pipe showing ring of thickness dr used as a control element for analysis of fluid flow.

average velocity. Consider a pipe of radius R and a control volume that is the walls of a hollow cylinder of thickness dr within the pipe, shown in Fig. 6.3.

The cross-sectional area of the ring of thickness dr is $dA = \pi[(r + dr)^2 - r^2)] = \pi(r^2 + 2rdr + (dr)^2 - r^2) = 2\pi rdr + (dr)^2$. Because dr is small $(dr)^2$ is negligible therefore, $dA = 2\pi rdr$. The volumetric rate of flow (volume/time) through the control volume is $VdA = 2\pi rVdr$. The total volume going through the pipe will be the integral of the volumetric rate of flow through the control volume from $r = 0$ to $r = R$. Substituting Equation (6.10) for V:

$$\bar{V}(\pi R^2) = \frac{(P_1 - P_2)}{4L\mu}(2\pi)\int_0^R (R^2 - r^2)r\,dr$$

Rearranging and integrating:

$$\bar{V} = \frac{(P_1 - P_2)}{2L\mu R^2}\left[\frac{R^2r^2}{2} - \frac{r^4}{4}\right]\Bigg|_0^R$$

Substituting limits, combining terms, and substituting ΔP for $(P_1 - P_2)$:

$$\bar{V} = \frac{(P_1 - P_2)\,R^2}{8L\mu} = \frac{\Delta PR^2}{8L\mu} \tag{6.11}$$

Equation (6.11) is the Poiseuille equation and can be used to determine the viscosity of a Newtonian fluid from pressure drop data when the fluid is allowed to flow through a tube or a capillary.

When Equation (6.11) is used to determine the viscosity of non-Newtonian fluids, the viscosity obtained will be an apparent viscosity. Thus a viscosity obtained from measurements using a single rate of flow is:

$$\mu_{app} = \frac{\Delta PR^2}{8L\bar{V}}$$

Equations (6.10) and (6.11) may be combined to obtain an expression for $V/\underline{V}$ which can be differentiated to obtain a rate of shear as a function of the average velocity.

$$V = \bar{V}\left[\frac{\Delta P}{4L\mu}\right](R^2 - r^2)\left[\frac{8L\mu}{\Delta PR^2}\right]$$

Simplifying:

$$V = 2\bar{V}\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{6.12}$$

Equation (6.12) represents the velocity profile of a Newtonian fluid flowing through a tube expressed in terms of the average velocity. The equation represents a parabola where the maximum velocity is 2 $\bar{V}$ at the center of the tube ($r = 0$). Differentiating Equation (6.12):

$$\frac{dV}{dr} = 2\bar{V}\left[\frac{2r}{R^2}\right]$$

The shear rate at the wall ($r = R$) for a Newtonian fluid is

$$-\frac{dV}{dr}\bigg|_w = \frac{4\bar{V}}{R} \tag{6.13}$$

6.2.1.3 Velocity Profile and Shear Rate for a Power Law Fluid

Equations (6.2) and (6.9) can be combined to give:

$$\frac{(P_1 - P_2)\,r}{2L} = K\left(-\frac{dV}{dr}\right)^n$$

Re-arranging and using ΔP for ($P_1 - P_2$):

$$\frac{dV}{dr} = \left[\frac{\Delta P}{2LK}\right]^{1/n}(r)^{1/n}$$

Integrating and substituting the boundary condition, $V = 0$ at $r = R$:

$$V = \left[\frac{\Delta P}{2LK}\right]^{1/n}\left[\frac{1}{(1/n) + 1}\right]\left[R^{(1/n)+1} - r^{(1/n)+1}\right] \tag{6.14}$$

Equation (7.14) represents the velocity profile of a power law fluid. The velocity profile equation will be more convenient to use if it is expressed in terms of the average velocity. Using a procedure similar to that used in the section "Derivation of the Poiseuille Equation," the following expression for the average velocity can be derived:

$$\bar{V}(\pi R^2) = \int_0^R 2\pi r\left[\frac{\Delta P}{2LK}\right]^{1/n}\left[\frac{n}{n+1}\right]\left[R^{(1/n)+1} - r^{(1/n)+1}\right]dr$$

Integrating and substituting limits:

$$\bar{V} = \left[\frac{\Delta P}{2LK}\right]^{1/n}[R]^{(n+1)/n}\left[\frac{n}{3n+1}\right] \tag{6.15}$$

The velocity profile in terms of the average velocity is

$$V = \bar{V}\left[\frac{3n+1}{n+1}\right]\left[1 - \left[\frac{r}{R}\right]^{(n+1)/n}\right] \tag{6.16}$$

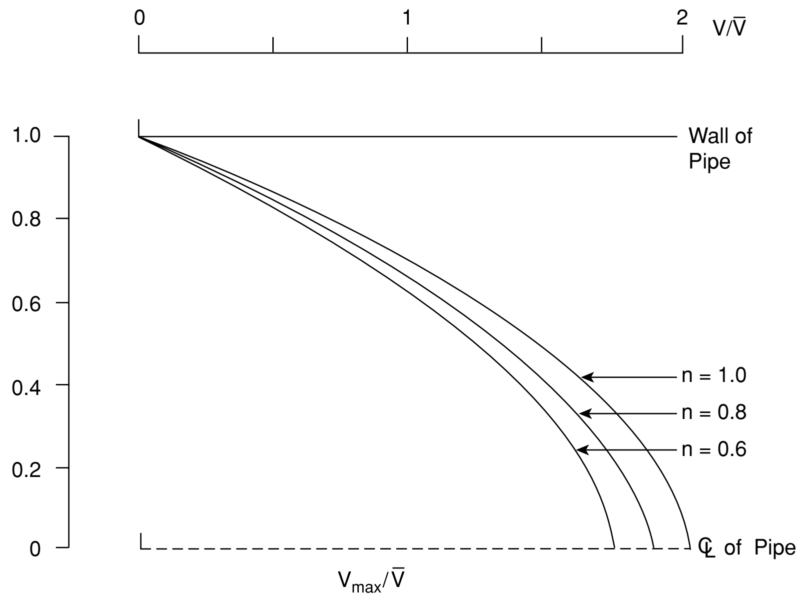

Figure 6.4 Plot of $V/\bar{V}$ as a function of position in the pipe for fluids with different values of the flow behavior index, n.

Figure 6.4 shows $V/\bar{V}$ as a function of r/R for various values of n. When $n = 1$, the velocity profile is parabolic and Equations (6.12) and (6.16) give the same results. The velocity profile flattens out near the center of the pipe as n decreases.

Differentiating Equation (6.16) and substituting $r = R$ for the shear rate at the wall:

$$-\frac{dV}{dr}\bigg|_w = \bar{V}\left[\frac{3n+1}{n+1}\right]\left[\frac{n+1}{n}\right][R]^{-(n+1)/n}[R]^{1/n}$$

Simplifying:

$$-\frac{dV}{dr}\bigg|_w = \bar{V}\left(\frac{3n+1}{n}\right)\left(\frac{1}{R}\right) \tag{6.17}$$

Equation (6.17) is a form of the Rabinowitsch-Mooney equation used to calculate shear rates for non-Newtonian fluids flowing through tubes. Converting into a form similar to Equation (6.13):

$$-\frac{dV}{dr}\bigg|_w = \frac{4\bar{V}}{R}\left[\frac{3}{4} + \frac{1}{4n}\right] \tag{6.18}$$

Equation (6.18) shows that the shear rate at the wall for a non-Newtonian fluid is similar to that for a Newtonian fluid except for the multiplying factor $(0.75 + 0.25/n)$.

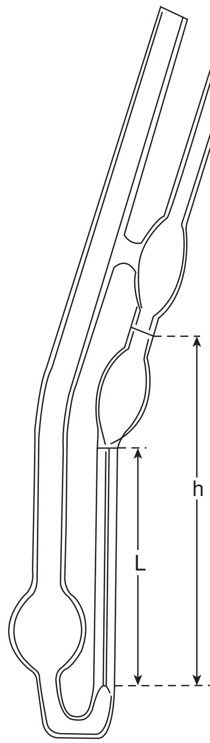

Figure 6.5 Diagram of a glass capillary viscometer showing the components and parameters used in fitting viscometer data to the Poiseuille equation.

6.2.1.4 Glass Capillary Viscometers

The simplest viscometer operate on gravity induced flow, and are commercially available in glass. Figure 6.5 shows a Cannon-Fenske type viscometer. These viscometers are available with varying size of the capillary. The size of capillary is chosen to minimize the time of efflux for viscous fluids. The Poiseuille equation (Eq. 6.11) is used to determine the viscosity in these viscometers. The pressure drop needed to induce flow is

$$\Delta P = \rho g h$$

Where ρ is the density of the fluid and h is the height available for free fall in the viscometer (Fig. 6.5). The viscosity is then calculated as using the above expression for ΔP in Equation (6.13) as follows:

$$\frac{\mu}{\rho} = \frac{ghR^2}{8L\bar{V}} \tag{6.19}$$

The ratio μ/ρ is the *kinematic viscosity*.

The viscosity is obtained from the time of efflux of the fluid through the viscometer. To operate the viscometer, fluid is pipetted into the large leg of the viscometer until the lowermost bulb is about half full. Fluid is then drawn into the small leg to about half of the uppermost bulb. When the suction on the small leg is released, fluid will flow and timing is started when the fluid meniscus passes through the first mark. When the fluid meniscus passes the second mark, timing is stopped and the efflux time is recorded. The lower portion of the small leg of the viscometer is a capillary of radius R and length L. If t_e is the time of efflux, the average velocity $\bar{V}$ will be L/t_e. Substituting in Equation (6.18):

$$\frac{\mu}{\rho} = \frac{ghR^2}{8L} t_e \tag{6.20}$$

For each viscometer, the length and diameter of capillary, and height available for free fall, are specific, therefore, these factors can be grouped into a constant , k_v, for a particular viscometer. The kinematic viscosity can then be expressed as:

$$\frac{\mu}{\rho} = k_v t_e \tag{6.21}$$

The viscometer constant is determined from the efflux time of a fluid of known viscosity and density.

Example 6.1. A glass capillary viscometer when used on a fluid with a viscosity of 10 centipoises allowed the fluid to efflux in 1.5 minutes. This same viscometer used on another fluid allowed an efflux time of 2.5 minutes. If the densities of the two fluids are the same, calculate the viscosity of the second fluid.

Solution:

Using Equation (6.21) to solve for the viscometer constant:

$$k_v = \frac{\mu}{\rho t_e} = \frac{10}{1.5\rho}$$

For the second fluid:

$$\mu = \rho \left[\frac{10}{1.5\rho} \right] (2.5) = 16.67 \text{ centipoises}$$

6.2.1.5 Forced Flow Tube or Capillary Viscometry

For very viscous fluids, gravity induced flow is not sufficient to allow measurement of viscosity. Forced flow viscometers are used for these fluids. In order to obtain varying flow rates in viscometers, some means must be provided to force a fluid through the viscometer at a constant rate. This may be obtained by using a constant pressure and measuring the flow rate that develops, or by using a constant flow rate and measuring the pressure drop over a length of test section. Flow rates through glass capillary viscometers may also be varied by applying pressure instead of relying solely on gravity flow.

The flow properties of a fluid are the constants in Equations (6.1) to (6.3), which can be used to characterize the relationship between the shear stress and the rate of shear. These are the viscosity, if the fluid is Newtonian, the flow behavior index, the consistency index, and the yield stress. Equation (6.3) is most general. If a fluid has a yield stress, the test is carried out at very high rates of shear in order that τ_o is very small in comparison to τ and Equation (6.3) simplifies to Equation (6.2). A fluid with properties that fits Equation (6.1) is a special case of a general class of fluids which fits Equation (6.2), when $n = 1$. Evaluation of τ_o in Equation (6.3), from data at low shear rates is simple, once n is established from data at high shear rates. The shear rate at the wall may be determined using Equations (6.13) or (6.18). These equations show that the shear rate is the product of some factor which is independent of the rate of flow, and another factor which is a function of the rate of flow, i.e the average velocity, $\bar{V}$, the volumetric rate of flow Q, or even the speed of a piston V_p which delivers fluid to the tube. In equation form:

$$\gamma_w = F_1\bar{V} = F_2Q = F_3V_p$$

The shear stress at the wall is determined by substituting R for r in Equation (6.9).

$$\tau_w = \frac{\Delta PR}{2L} \tag{6.22}$$

Equation (6.22) shows that τ_w is a product of a factor which is independent of the rate of flow, and the pressure drop. In equation form: $\tau_w = F_{p1}(\Delta P) = F_{p2}$(height of manometer fluid) $= F_{p3}$(transducer output). Substitution of any of the above expressions for τ_w and Δ_w into Equation (6.2):

$$PF_{p1} = [\bar{V}F_1]^n$$

P and $\bar{V}$ are, respectively, measurements from which pressure or velocity can be calculated. Taking logarithms of both sides:

$$\log P + \log F_{p1} = n \log \text{overlinbe}V + n \log F_1$$

Rearranging:

$$\log P = n \log \bar{V} + (n \log F_1 - \log F_{p1})$$

Thus, a log-log plot of any measure of pressure against any measure of velocity can give the flow behavior index n for a slope.

The shear rate can then be calculated using either Equation (6.13) or (6.18), and the shear stress can be calculated using Equation (6.22). Taking the logarithm of Equation (6.2): $\log \tau_w = \log K + n \log \gamma_w$.

The intercept of a log-log plot of τ_w against γ_w will give K.

Example 6.2. A tube viscometer having an inside diameter of 1.27 cm and a length of 1.219 m is used to determine the flow properties of a fluid having a density of 1.09 g/cm$^{3.}$ The following data were collected for the pressure drop at various flow rates measured as the weight of fluid discharged from the tube. Calculate the flow behavior and consistency indices for the fluid.

Data with pressure drop (in kPa)	
$(P_1 - P_2)$	*Flow rate (g/s)*
19.197	17.53
23.497	26.29
27.144	35.05
30.350	43.81
42.925	87.65

Solution:

Figure 6.6 shows a plot of log ΔP against log (mass rate of flow). The slope, $n = 0.5$, indicates that the fluid is non-Newtonian. Equation (6.18) will be used to solve for γ_w, and Equation (6.22) for τ_w.

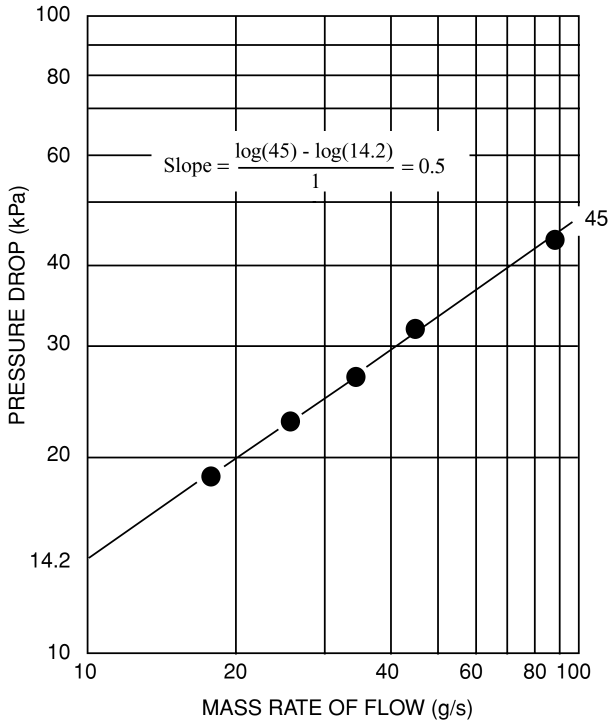

Figure 6.6 Log-log plot of pressure drop against mass rate of flow to obtain the flow behavior index from the slope.

Solving for τ_w: R = 0.5(1.27 cm)(0.01 m/cm) = 0.00635 m; L = 1.219 m.

$$\tau_w = [0.00635(0.5)/1.219]\,\Delta P = 0.002605\,\Delta P\,\text{Pa}$$

The average velocity $\bar{V}$ in m/s can be calculated by dividing the mass rate of flow by the density and the cross-sectional area of the tube. Let q = mass rate of flow in g/s.

$$\begin{aligned}\bar{V} &= q\frac{g}{s}\,\frac{cm^3}{1.09\,g}\,\frac{m^3}{(100)^3\,cm^3}\,\frac{1}{\pi(0.00635)^2\,m^2}\\ &= 0.007242\,q\ \text{m/s}\end{aligned}$$

Equation (6.18) is used to calculate the shear rate at the wall. n = 0.5.

$$\begin{aligned}\gamma_w &= \frac{4\bar{V}}{R}\left(0.75 + \frac{0.25}{n}\right) = \frac{4(0.007242\,q)}{0.00635}(1.25)\\ &= 5.7047\,q\end{aligned}$$

The shear stress and shear rates are

τ_w (Pa)	γ_w (1/s)
50.008	99.966
61.209	149.92
70.710	199.87
79.062	249.83
111.82	499.83

Equation (6.2) will be used to determine K. The yield stress τ_o is assumed to be much smaller than the smallest value for τ_w measured. Thus, Equation (6.3) reduces to the same form as Equation (6.2), the logarithm of which is as follows:

$$\log \tau_w = \log K + n \log \gamma_w$$

A log-log plot of shear stress against the shear rate is shown in Fig. 6.7. The intercept is the consistency index,

$$k = 5\ \mathrm{Pa \cdot s^n}.$$

Example 6.3. A fluid induces a pressure drop of 700 Pa as it flows through a tube having an inside diameter of 0.75 cm and a length of 30 cm at a flow rate of 50 cm^3/s.

(a) Calculate the apparent viscosity defined as the viscosity of a Newtonian fluid that would exhibit the same pressure drop as the fluid at the same rate of flow through this tube.
(b) This same fluid flowing at the rate of 100 cm^3/s through a tube 20 cm long and 0.75 cm in inside diameter, induces a pressure drop of 800 Pa. Calculate the flow behavior and consistency indices for this fluid.
(c) What would be the shear rates at 50 and 100 cm^3/s?

Solution:

Equation (6.11) will be used to calculate the apparent viscosity from the pressure drop of a fluid flowing through a tube.

(a) $R = (0.0075)(0.5) = 0.00375\,\mathrm{m}$; $L = 0.3\,\mathrm{m}$; $\Delta P = 700$ Pa.

$$\bar{V} = 50\frac{\mathrm{cm^3}}{\mathrm{s}}\,\frac{1\,\mathrm{m^3}}{(100)^3\,\mathrm{cm^3}}\,\frac{1}{\pi(0.00375)^2} = 1.1318\,\mathrm{m/s}$$

$$\mu_{app} = \frac{\Delta P R^2}{8L\bar{V}}$$

$$\mu_{app} = \frac{700(0.00375)^2}{8(0.3)(1.1318)} = 0.003624\,\mathrm{Pa \cdot s}$$

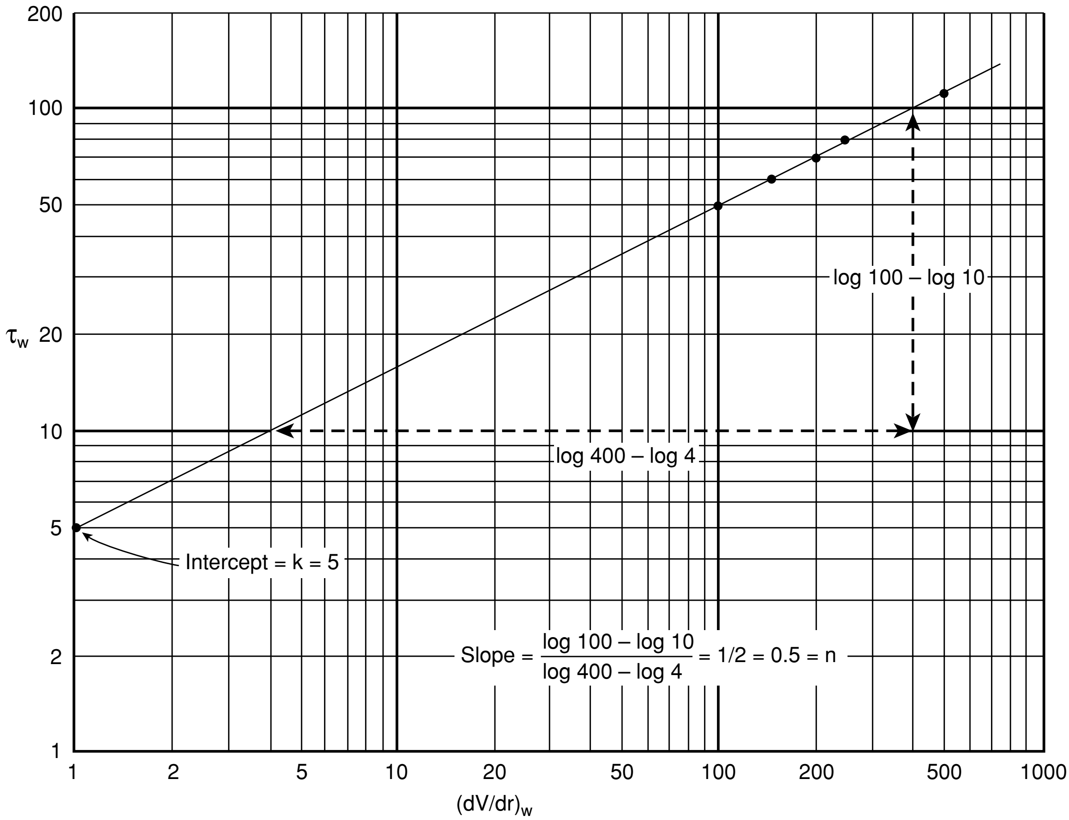

Figure 6.7 Log-log plot of shear stress against shear rate to obtain the consistency index from the intercept.

(b) $\bar{V} = 100\frac{cm^3}{s}\frac{1\ m^3}{(100)^3\ cm^3}\frac{1}{\pi(0.00375)^2} = 2.2635\ m/s$

$$\mu_{app} = \frac{800(0.00375)^2}{8(0.2)(2.2635)} = 0.003106\,Pa\cdot s$$

Equation (6.6) will be used to solve for K and n.

Let $\varphi = 0.75 + 0.25/n$

Equation (6.18): $\gamma_w = \dfrac{4\bar{V}}{R}(\phi)$

Equation (6.6) : $\mu_{app} = K(\gamma)^{n-1}$

Substituting ϕ : $\mu_{app} = K\left[\dfrac{4\phi}{R}\right]^{n-1}\bar{V}^{(n-1)}$

Using the subscripts 1 and 2 for the velocity and apparent viscosity at 50 and 100 cm^3/s, respectively:

$$\frac{\mu_{app}1}{\mu_{app}2} = \frac{(K(4\phi/R)^{n-1}\bar{V}_1)^{n-1}}{(K(4\phi/R)^{n-1}\bar{V}_2)^{n-1}} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

$$n = 1 + \frac{\log(1.16674)}{\log(0.5009)}$$

$$\log(1.16674) = (n-1)\log(0.5009)$$

$$\log\left(\frac{0.003624}{0.003106}\right) = (n-1)\log\left(\frac{1.1318}{2.2635}\right) = 0.777$$

$$\phi = 0.75 + \frac{0.25}{0.777} = 1.072$$

Using Equation (6.6) on either of the two apparent viscosities:

$$\gamma_w = \frac{4\,(1.1317)}{0.00375}(1.072) = 1294\ s^{-1}$$

Using Equation (6.6):

$$K = \frac{0.003624}{(1294)^{0.777-1}} = 0.0178\ Pa \cdot s^n$$

(c) At 50 cm^3/s, $\gamma_w = 1294\ s^{-1}$

At 100 cm^3/s, $\gamma_w = \dfrac{4(2.2635)}{0.00375}(1.072) = 2414\ s^{-1}$

6.2.1.6 Evaluation of Wall Effects in Tube Viscometry

The equations derived in the previous sections for evaluating flow properties of fluids by tube or capillary viscometry are based on the assumption that there is zero slip at the wall. This condition may not always be true for all fluids, particularly suspensions.

To determine if slip exists at the wall, it will be necessary to conduct experiments on the same fluid using viscometers of different radius. Kokini and Plotchok (1987) defined a parameter, $ß_c$, which can be used to correct for slip at the tube wall. $ß_c$ is the slope of a plot of $Q/(\pi R^3 \tau_w)$ again $1/R^2$ obtained on the same fluid using different viscometers of varying radius at flow rates which would give a constant shear stress at the wall, τ_w. Q = the volumetric rate of flow, and R is the viscometer radius. With $ß_c$ known, a corrected volumetric flow rate Q_c is calculated as follows:

$$Q_c = Q - \pi R \tau_w ß_c \qquad (6.23)$$

The corrected flow rate Q_c is then used instead of the actual flow rate in determining the mean velocity and shear rates in the determination of the flow behavior index and the consistency index of fluids.

The experiments will involve a determination of pressure drop at different rates of flow on at least four tubes, each with different radius. The value of $ß_c$ may vary at different values of τ_w, therefore, for each tube, data are plotted as log Q against log τ_w. The four linear plots representing data obtained on the four tubes are drawn on the same graph, and at least four values of τ_w are selected. Points on each line at a constant value of τ_w, are then selected. At each value of τ_w, a plot is made of $Q/(\pi R^3 \tau_w)$ against $1/R^2$. The plot will be linear with a slope, $ß_c$. The value of $ß_c$ may be different at different τ_w, therefore, plots at 4 different values of τ_w are needed. After determining $ß_c$ values at each τ_w, Equation (6.23) is used to determine Q_c, which is then used to determine Q_w. An example is given in Kokini and Plotchok's (1987) article.

6.2.1.7 Glass Capillary Viscometer Used as a Forced Flow Viscometer

Fluids that exhibit non-Newtonian behavior but have low consistency indices are usually difficult to study using tube viscometers because of the low pressure drop. A glass capillary viscometer may be used as a forced flow viscometer by attaching a constant pressure source to the small leg of the viscometer. A simple set-up is shown in Fig. 6.8.

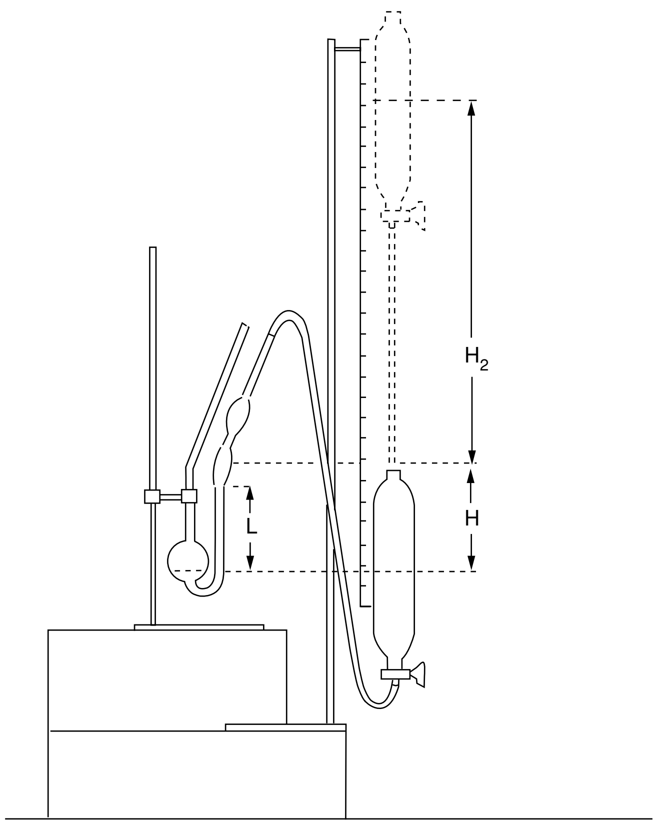

Figure 6.8 Diagram of a system used to adapt a glass capillary viscometer for forced flow viscometry.

The pressure source is a water column of movable height. To operate, the fluid is pipetted into the large leg of the viscometer. The fluid level in the u-tube is made level with the fluid in the large bulb of the viscometer by adjusting the height of the dropping funnel and the stopcock is shut-off. The viscometer is disconnected from the pressure source and fluid is drawn to the top of the second bulb on the small viscometer leg by suction. After reconnecting the viscometer to the pressure source, the height of the dropping funnel is raised and pressure is applied by opening the stopcock. The time of efflux is then measured as the time for the fluid meniscus to pass through the first and second mark on the viscometer. Pressure is discontinued by closing the stopcock and the difference in fluid level between the dropping funnel and the large bulb of the viscometer is measured. The flow behavior and consistency indices will be calculated from the efflux times at different levels of fluid in the pressure source.

In a capillary viscometer, the volume of fluid which passes through the capillary is the volume which fills the section between the two marks on the viscometer. Two other critical factors are the length of the capillary (L) and the height available for free fall (H). These measurements are made on the viscometer as indicated in Fig. 6.8. The radius of the capillary may be calculated from the viscometer constant using Equation (6.19). When used as a forced flow viscometer, the pressure drop will be the pressure equivalent to the height of the forcing fluid as shown in Fig. 6.8 and the pressure equivalent to the height available for free fall of the fluid in the viscometer. The calculations are shown in the following example.

Example 6.4. A Cannon-Fenske type glass capillary viscometer has a height available for free fall of 8.81 cm and a length of capillary of 7.62 cm. When used on distilled water at 24°C, an efflux time of 36 seconds was measured. The volume of fluid which drains between the two marks on the viscometer was 3.2 cm^3.

(a) Calculate the radius of the capillary.
(b) When used as a forced flow viscometer on a concentrated acid whey containing 18.5% total solids at 24°C, the following data were collected:

Height of (H_2) forcing fluid (cm)	*Efflux time (t_e) (s)*
0	92
7.1	29
20.8	15
34.3	9

The whey has a density of 1.0763 g/cm^3. Calculate the flow behavior and consistency indices.

Solution:

(a) At 24°C, the density and viscosity of water are 947 kg/m^3 and 0.00092 Pa · s, respectively. Using Equation (6.11):

$$\mu = \frac{\Delta PR^2}{8L\bar{V}}; \Delta P = \rho gH$$

$$\mu = \frac{\rho gHR^2\pi R^2 t_e}{3.2 \times 10^{-6}(8)L} \quad R^4 = \frac{(3.2 \times 10^{-6})\, 8\, L\mu}{\rho gH\pi t_e}$$

$$\bar{V} = \frac{\text{Volume}}{\pi}R^2 t_e$$

Substituting known values and solving for R:

$$R = \left[\frac{0.00092\,(3.2 \times 10^{-6})(8)(0.0762)}{997\,(9.8)(0.0881)(\pi)(36)}\right]^{0.25}$$
$$= 3.685 \times 10^{-4}\ \text{m}$$

(b) The total pressure forcing the fluid to flow is: $\Delta P = g(\rho_1\, H + \rho_2\, H_2)$

$$\Delta P = 9.8\,[(1076.3)(0.0881) + (997)(H_2)] = 9.8(94.822 + 997H_2)\ \text{Pa}$$

Let q = volumetric rate of flow.

$$q = \frac{3.2}{t_e}\ \text{cm}^3/\text{s}$$

The pressure inducing flow, and the volumetric rate of flow calculated using the above equations are

ΔP *(Pa)*	*q (cm^3/s)*
929	0.0564
1623	0.110
2961	0.213
4280	0.355

A log-log plot of ΔP against q shown in Fig. 6.9 gives n = 0.84.

The consistency index will be calculated from the rates of shear and the shear stress.

$$\bar{V} + \frac{q \times 10^{-6}}{\pi(R^2)} = 2.3466\,q$$

$$\gamma_w = \frac{4\bar{V}}{R}\left[0.75 + \frac{0.25}{n}\right] = 26699\,q$$
$$= \frac{4(2.3466)\,q}{3.683 \times 10^{-4}}\left[0.75 + \frac{0.25}{0.84}\right]$$

The shear stress is calculated from ΔP. Substituting known values in Equation (6.22):

$$\tau_w = \frac{\Delta PR}{2L} = \frac{\Delta P(3.683 \times 10^{-4})}{2(0.00762)} = 24.1666 \times 10^{-4}\Delta P$$

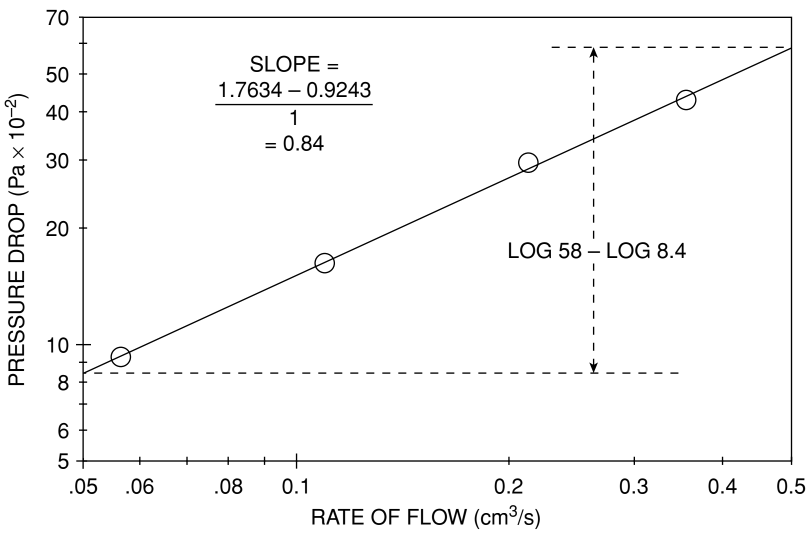

Figure 6.9 Log-log plot of pressure drop against the rate of flow to determine the flow behavior index, n, from the slope.

The shear stress and shear rates are

τ_w *(Pa)*	γ_w *(1/s)*
2.245	1505
3.922	2936
7.155	5687
10.343	9478

Figure 6.10 shows a log-log plot of shear stress against shear rate. A point on the line can be used to calculate K. The first data point (2.245,1505) falls directly on the line.

$$K = \frac{\tau_w}{(\gamma_w)^n} = \frac{2.245}{(1505)^{0.84}} = 0.0048\,\text{Pa} \cdot \text{s}^n$$

6.2.2 Effect of Temperature on Rheological Properties

Temperature has a strong influence on the resistance to flow of a fluid. It is very important that temperatures be maintained constant when making rheological measurements. The flow behavior index, n, is relatively constant with temperature, unless components of the fluid undergo chemical changes at certain temperatures. The viscosity and the consistency index on the other hand, are highly temperature dependent.

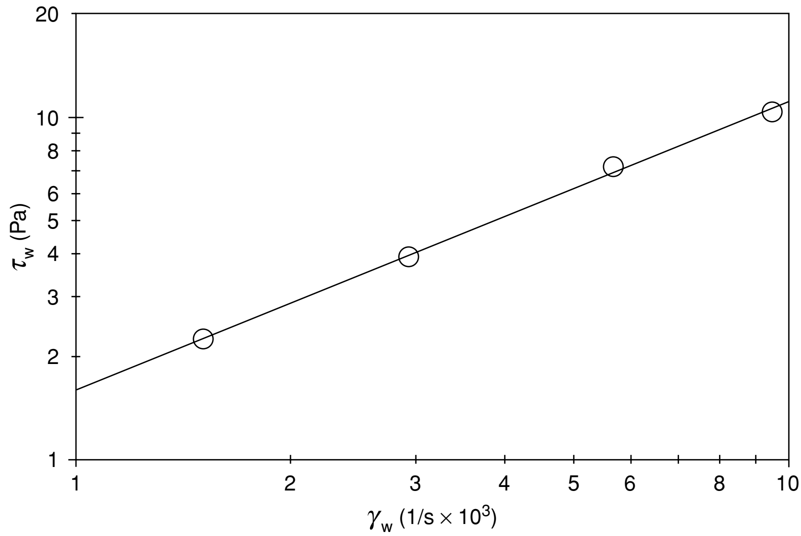

Figure 6.10 Log-log plot of shear stress against shear rate to determine a point on the line that can be used to calculate the consistency index, K.

The temperature dependence of the viscosity and the consistency index can be expressed in terms of the Arrhenius equation:

$$\ln\left(\frac{\mu}{\mu_1}\right) = \frac{E_a}{R}\left(\frac{1}{T} - \frac{1}{T_1}\right) \tag{6.24}$$

where μ = the viscosity at absolute temperature T; μ_1 = viscosity at temperature T_1; E_a = the activation energy, J/gmole; R = the gas constant, 8.314 J/(gmole K); and T is the absolute temperature.

Equation (6.24) is useful in interpolating between values of μ at two temperatures. When data at different temperatures are available, Equation (6.24) may be expressed as:

$$\ln \mu = A + \frac{B}{T} \tag{6.25}$$

where the constants B and A are slope and intercept respectively of the plot of $\ln(\mu)$ against 1/T. The same expressions may be used for the temperature dependence of the consistency index, K.

There are other expressions for the dependence of the viscosity or consistency index on temperature such as the Williams-Landel-Ferry (WLF) equation and the Fulcher equation (Rao et al. 1987), but these also have limitations for general use and determination of the parameters from experimental data requires specialized curve-fitting computer programs.

The WLF equation however, has implications in the flow behavior of polymer solutions and the viscoelastic properties of gels, therefore, and the basis for the model will be briefly discussed. The model is based on the concept of a free volume. Any polymer, by virtue of the size of the molecule, contains a free volume. The free volume may include the space within the helix in a protein or starch molecule, or the space formed when sections of a random coil mesh. The magnitude of the free volume is temperature dependent. At a specific temperature called the glass transition temperature, the free volume is essentially zero, and this will be manifested by a drastic change in the thermophysical properties of the material. The free volume is best manifested by the specific volume. A plot of the specific volume with temperature above the glass transition temperature is continuous and the slope defines the thermal expansion coefficient. The point in the curve where a discontinuity occurs, is the glass transition temperature. The WLF model relates the ratio, μ/μ_g, where μ = the viscosity at temperature T and μ_g = the viscosity at the glass transition temperature, T_g, to a function of T − Tg. The Arrhenius equation is the simplest to use and is widely applied in expressing the temperature dependence of a number of factors.

Example 6.5. The flow behavior index of applesauce containing 11% solids is 11.6 and 9.0 $Pa \cdot s^n$, at 30°C and 82°C respectively. Calculate the flow behavior index at 50°C.

Solution:

Using Equation (6.24): T = 303 K; T_1 = 355 K

$$\frac{E_a}{R} = \frac{\ln(11.6/9.0)}{1/303 - 1/355} = 524.96$$

$$K = 11.6[e]^{524.96(1/323-1/303)}$$

Using Equation (6.24) with k_1 = 11.6, T_1 = 303 K, and T = 323 K.

$$K = 11.6(.898) = 10.42\,Pa \cdot s^n$$

6.2.3 Back Extrusion

Back extrusion is another method for evaluating the flow properties of fluids. It is particularly useful with materials that have the consistency of a paste or suspensions that have large particles suspended in them, since the suspended solids tend to intensify wall effects when the fluid is flowing in a small tube. When a rotational viscometer is used, pulsating torque readings are usually exhibited when a lumpy fluid is being evaluated.

The key to accurate determination of fluid flow properties using back extrusion is the assurance of annular flow, i.e. the plunger must remain in the center of the larger stationary cylinder all throughout the test. Figure 6.11 shows a back extrusion device designed for use on an Instron universal testing machine. A stanchion positioned at the center of the large cylinder which is machined to a close tolerance to fit an opening at the center of the plunger, guides the movement of the plunger and ensures concentric positioning of the plunger and outer cylinder at all times during the test.

The equations that govern flow during back extrusion have been derived by Osorio and Steffe (1987).

Let F_t = the force recorded at the maximum distance of penetration of the plunger L_p. The height of the column of fluid which flowed through the annulus, L is

$$L = \frac{A_p L_p}{A_a}$$

where A_p is the area of the plunger which displaces fluid, and A_a is the cross-sectional area of the annulus.

$$A_p = \pi(R_p^2 - R_s^2); \quad A_a = \pi(R_c^2 - R_p^2)$$

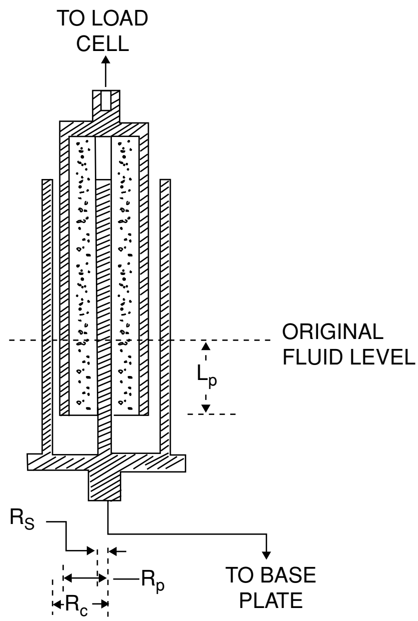

Figure 6.11 Diagram of a back extrusion cell suitable for measuring flow properties of fluids with lumpy or paste-like consistency.

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where R_c is the radius of the cylinder, R_p is the radius of the plunger, and R_s is the radius of the stanchion which positions the plunger in the center of the cylinder. L_p cannot easily be measured on the plunger and cylinder assembly itself since the fluid level in the cylinder is not visible outside, but it can easily be read from the force tracing on the Instron chart, by measuring the distance from the initiation of the rise in force and the point of maximum force on the chart. Figure 6.12 shows a typical back extrusion force-displacement curve.

Part of the total force F_t counteracts the force of gravity on the mass of fluid in the annulus.

Force of gravity on annular fluid $= \rho g\, L\pi({R_c}^2 - {R_p}^2)$

The net force, F_n, which is exerted by the fluid to resist flow through the annulus, is the total force corrected for the force of gravity.

$$F_n = F_t - \rho g L\pi(R_c^2 - R_p^2)$$

where ρ = density of the fluid.

The flow behavior index, n, is the slope of a log-log plot of F_n/L, against the velocity of the plunger, V_p.

Determination of the shear rate at the plunger wall and the shear stress at this point involves a rather complex set of equations which were derived by Osorio and Steffe (1987). The derivation is based on a dimensionless radius, λ, which is the ratio r_m/R_c , where r_m = radius of a point in the flow stream where velocity is maximum. The deferential momentum and mass balance equations were solved simultaneously to obtain values of λ for different values for the flow behavior index of fluids, and

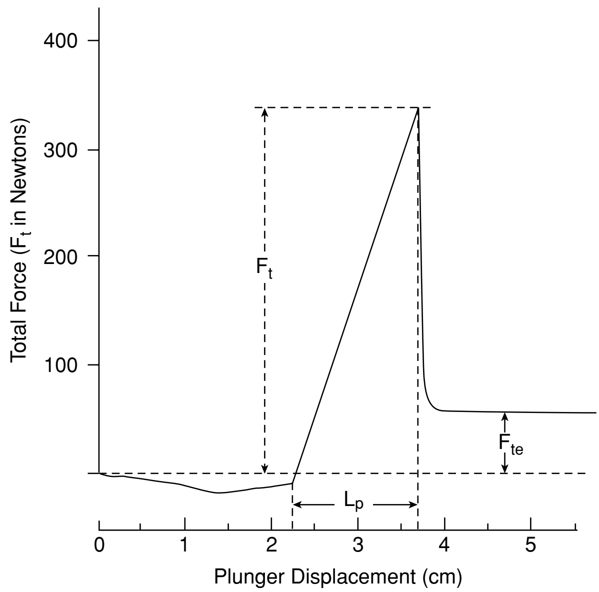


Figure 6.12 Typical force deformation curve obtained for back extrusion showing how total force, plunger displacement, and yield force are obtained.

different annular gaps. The authors presented results of the calculations as a Table of λ for different n and annular gap size expressed as σ, the ratio of plunger to cylinder radius (R_p/R_c). Table 6.1 shows these values. Figure 6.13 related the value of σ to a parameter θ for fluids with different n. The parameter θ is used to calculate the shear rate at the wall from the plunger velocity, V_p.

Table 6.1 Values of λ for Different Values of σ and *n*.

	n									
σ	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1.0*
0.1	0.4065	0.4889	0.5539	0.6009	0.6344	0.6586	0.6768	0.6907	0.7017	0.7106
0.2	0.5140	0.5680	0.6092	0.6398	0.6628	0.6803	0.6940	0.7049	0.7138	0.7211
0.3	0.5951	0.6313	0.6587	0.6794	0.6953	0.7078	0.7177	0.7259	0.7326	0.7382
0.4	0.6647	0.6887	0.7068	0.7206	0.7313	0.7399	0.7469	0.7527	0.7575	0.7616
0.5	0.7280	0.7433	0.7547	0.7636	0.7705	0.7761	0.7807	0.7846	0.7878	0.7906
0.6	0.7871	0.7962	0.8030	0.8082	0.8124	0.8158	0.8186	0.8209	0.8229	0.8246
0.7	0.8433	0.8480	0.8516	0.8544	0.8566	0.8584	0.8599	0.8611	0.8622	0.8631
0.8	0.8972	0.8992	0.9007	0.9019	0.9028	0.9035	0.9042	0.9047	0.9052	0.9055
0.9	0.9493	0.9498	0.9502	0.9504	0.9507	0.9508	0.9510	0.9511	0.9512	0.9513

From: Osorio and Steffe (1987); used with permission.

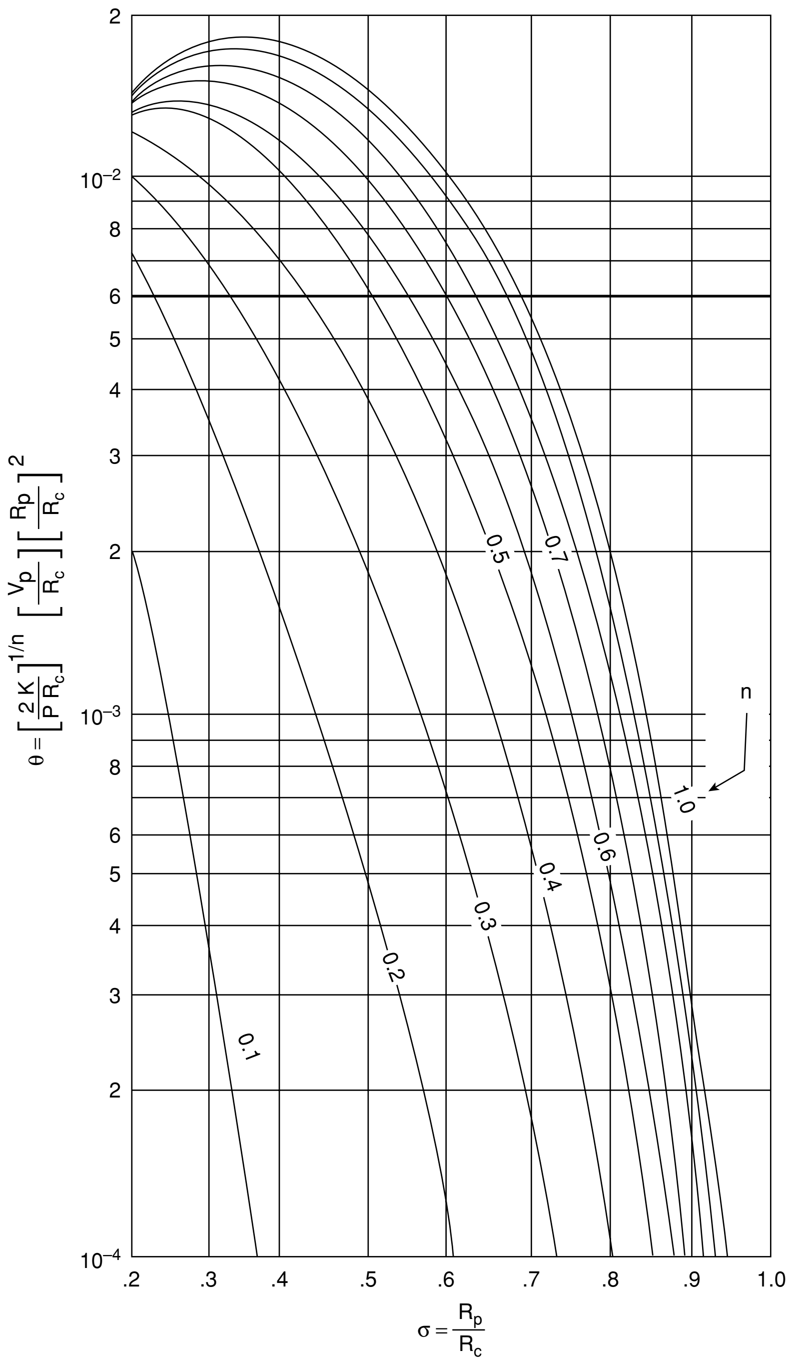


Figure 6.13 Graph of the parameter θ used to determine the shear rate from the plunger velocity. (Source: Osorio, F. A. and Steffe, J. F. 1987. Back extrusion of power law fluids. J. Tex. Studies. 18:43. Used with permission.)

Let : $\sigma = R_p/R_c$; $\sigma_s = R_s/R_c$; $\lambda = r_m/R_c$

The parameter θ in Fig. 6.12 is a dimensionless parameter which satisfies the expressions for the integral of the dimensionless point velocity of fluid through the annulus and the quantity of fluid displaced by the plunger. The terms V_p, P, and K in the expression of θ in Fig. 6.13 are V_p = plunger velocity, P = pressure drop per unit distance traveled by the fluid, and k = the fluid consistency index.

Knowing the flow behavior index of the fluid, and the dimensionless plunger radius, σ, a value for λ is obtained from Table 6.1. Figure 6.13 is then used to obtain a value for θ. The shear rate at the plunger wall is calculated using Equation (6.26).

$$-\left.\frac{dV}{dr}\right|_{R_p} = \frac{V_p}{R_c\theta}(\sigma^2 - \sigma_s{}^2)\left[\frac{\lambda^2}{\sigma} - \sigma\right]^{\frac{1}{n}} \tag{6.26}$$

The shear stress at the plunger wall is calculated from the net force, F_n, exerted on the base of the column of fluid to force it up the annulus.

The shear stress at the wall, τ_w, is calculated using Equation (6.27). Equations (6.26) and (6.27) are derived to account for the fact that part of the plunger area being occupied by the stanchion which positions the plunger in the center of the concentric cylinder, does not displace fluid up through the annular gap.

$$\tau_w = \frac{F_n}{2\pi L R_p}\left(\frac{\lambda^2 - \sigma^2}{\lambda^2 - \sigma^2}\right) \tag{6.27}$$

The consistency index of the fluid can then be determined from the intercept of a log-log plot of shear stress and shear rate.

The yield stress can be calculated from the residual force after the plunger has stopped, F_{ne} corrected for the hydrostatic pressure of the height of fluid in the annulus.

$$\tau_o = \frac{(F_{ne})}{2\pi(R_p + R_c)L} \tag{6.28}$$

where $F_{ne} = F_{te} - \rho g L\pi(R_c^2 - R_p^2)$ and F_{te} = the residual force after the plunger has stopped.

Example 6.6. The following data were collected in back extrusion of a fluid through a device similar to that shown in Fig. 6.8. The diameters were: 2.54 cm for the stanchion, 6.64 cm for the plunger, and 7.62 cm for the outer diameter. The fluid had a density of 1.016 g/cm^3. The total force, the distance penetrated by the plunger, and the plunger speed at each of the tests are as follows:

Plunger speed, V_p (mm/min)	*Penetration depth, L_p (cm)*	*Total force, F_p (N)*
50	15.2	11.44
100	13.9	12.16
150	16.0	15.36
200	13.2	13.60

Calculate the flow behavior and consistency indices for this fluid.

Solution:

The flow behavior index is calculated by determining the ratio F_n/L and plotting against V_p.

$$L = L_p \frac{[(0.0332)^2 - (0.0127)^2]}{[(0.0381)^2 - (0.0332)^2}$$
$$= 2.6905 L_p$$
$$F_n = F_t - 1016(9.8)(2.6905 L_p)(\pi)[(0.0381)^2 - (0.0332)^2]$$
$$= F_t - 29.4028\, L_p$$

Values of F_n, L, and F_n/L at different V_p are

V_p (mm/min)	L (m)	F_n (N)	F_n/L (N/m)
50	0.408	6.98	17.1
100	0.374	8.08	21.59
150	0.430	10.66	24.79
200	0.355	9.72	27.37

A log-log plot of F_n/L against V_p is shown in Fig. 6.14. The slope is 0.33, the flow behavior index, n. The value of λ for $\sigma = 0.871$, $n = 0.33$ is obtained by interpolation from Table 6.1.

The value of λ for $n = 3$, $\sigma = 0.871$ is

$$\lambda = 0.9007 + \frac{(0.9504 - 0.9019)(0.071)}{0.1} = 0.9351$$

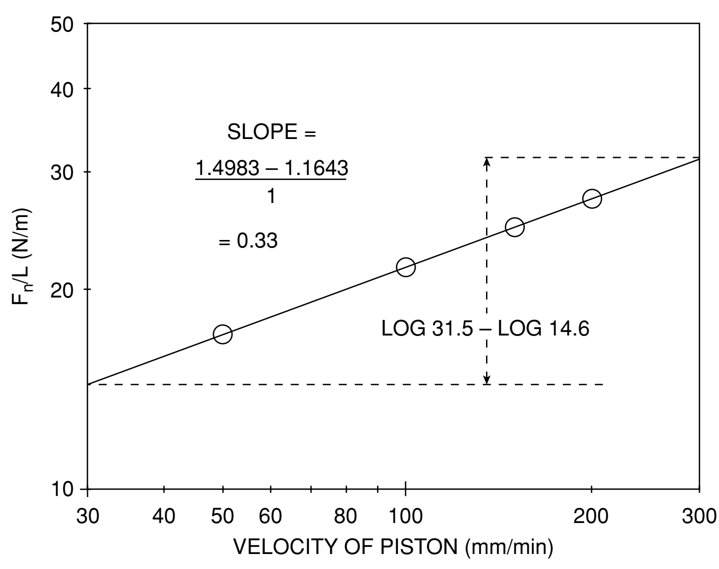


Figure 6.14 Log-log plot of corrected force to extrude fluid through an annular gap, against piston velocity, to obtain the flow behavior index, n, from the slope.

The value of λ for $n = 0.4$, $\sigma = 0.871$ is

$$\lambda = 0.9019 + \frac{(0.9504 - 0.9019)(0.071)}{0.1} = 0.9363$$

The falue of λ for $n = 0.33$ is obtained by interpolation:

$$\lambda = \frac{0.9351 + (0.9363 - 0.9351)(0.03)}{0.01} = 0.9354$$

The value of θ is obtained from Fig. 6.13. The curves for $n = 0.3$ and $n = 0.4$ do not intercept $\sigma = 0.871$ in Fig. 6.13. However, the curves are linear at low values of θ, and may be represented by the following equations:

$$n = 0.3 : \log\theta = 1.48353 - 7.5767\sigma$$
$$n = 0.4 : \log\theta = 1.9928 - 7.4980\sigma$$

Thus, at $n = 0.3$, $\theta = 7.665 \times 10^{-6}$; at $n = 0.4$, $\theta = 2.8976 \times 10^{-5}$. At $n = 0.33$, $\theta = 1.4058 \times 10^{-5}$ by interpolation. Knowing the value of n, θ, and λ, τ_w and γ_w can be calculated. Using Equation (6.26):

$$\sigma = 0.871,\ \lambda = 0.9354,\ \sigma_s = 0.3333,\ 1/n = 3.$$

$$\gamma_w = \frac{1}{0.381(1.4058 \times 10^{-5})}[(0.871)^2 - (0.3333)^2]\left[\frac{(0.9354)^2}{0.087}\right]^3\left(\frac{1}{60{,}000}\right)V_p$$
$$= 0.04796\ V_p;$$

where V_p is in mm/min.

Using Equation (6.27):

$$\tau_w = \frac{(0.9354)^2 - (0.871)^2}{2\pi(0.0332)[(0.9354)^2 - (0.3333)^2]}\left(\frac{F_n}{L}\right)$$
$$= 0.73(F_n/L)$$

The values of the shear stress and shear rates are

τ_w *(Pa)*	λ_w
12.48	2.398
15.76	4.796
18.09	7.194
19.98	9.592

The consistency index, K is determined from the intercept of the log-log plot in Fig. 6.15.

$$K = 9.2\,\mathrm{Pa} \cdot s^n$$

6.2.4 Determination of Rheological Properties of Fluids Using Rotational Viscometers

The most common rotational viscometer used in the food industry is the concentric cylindrical viscometer. The viscometer shown in Fig. 6.l6 consists of concentric cylinders with fluid in the annular

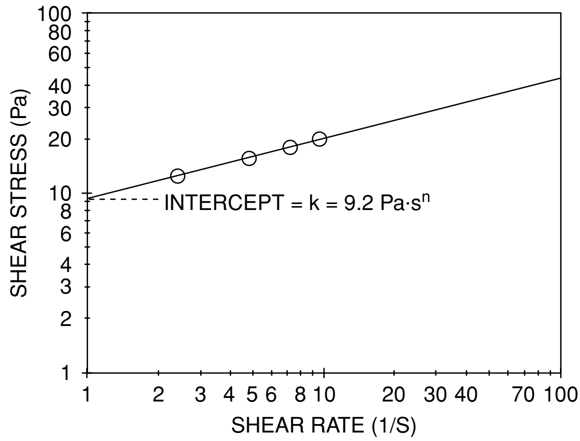


Figure 6.15 Log-log plot of shear stress against shear rate to obtain the consistency index, K, from the intercept.

space. This system is also referred to as a couette. The torque needed to rotate one of the cylinders is measured. This torque is proportional to the drag offered by the fluid to the rotation of the cylinder.

If the outer cup is stationary and if the torque measured is A, the force acting on the surface of the inner cylinder to overcome the resistance to rotation will be A/R.

The shear stress τ_w at the wall is

$$\tau_w = \frac{A}{R}\frac{1}{2\pi RL} = \frac{A}{R^2(2\pi L)} \tag{6.29}$$

For a Newtonian fluid in narrow gap viscometers where the rotating cylinder has a radius, R and the

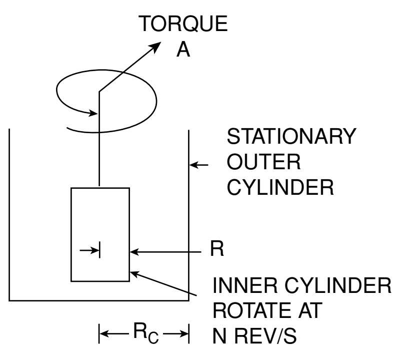


Figure 6.16 Schematic diagram of a rotational viscometer.

cup radius is R_c, the shear rate at the wall at a rotational speed of N rev/s is

$$\gamma_w = \frac{4\pi N R_c}{(R_c^2 - R^2)} \tag{6.30}$$

If the fluid is non-Newtonian with a flow behavior index, n, the shear rate at the wall has been derived by Kreiger and Maron (1954) for a couette type viscometer ($R_c/R < 1.2$) as follows:

$$\gamma_w = N\left[\frac{4\pi R_c^2}{(R_c^2 - R^2)} + 2\pi\left[1 + \frac{2}{3}\ln\left(\frac{R_c}{R}\right)\right]\left(\frac{1-n}{n}\right) + \frac{2\pi}{3}\ln\left(\frac{R_c}{R}\right)\left[\frac{1-n}{n}\right]^2\right] \tag{6.30a}$$

Equation (6.30) is a special case of Equation (6.30a) when the flow behavior index = 1. The factor in brackets which is a multiplier for N in Equation (6.30a), is easily calculated using a spreadsheet program.

Example 6.7. A rotational viscometer with a spring constant equivalent to 673.7 dyne ≅ cm full scale is used on a narrow gap viscometer with a rotating cylinder diameter of 29.5 mm and a cup diameter of 32 mm. The cylinders are 44.5 mm high. Assume end effects are negligible. This couette type viscometer is used to measure the flow properties of whole eggs and the data are shown in the spreadsheet (Figure 6.17) with N in rev/min entered in cells A4 to A7 and torque in % of full scale entered in cells B4 to B7. Calculate the flow behavior and consistency indices in SI units. The term in brackets to multiply N to obtain the shear rate in Equation (6.30a) has a value of 87.2 when n = 0.66 and is entered in cell A12 in Figure 6.17.

Solution:

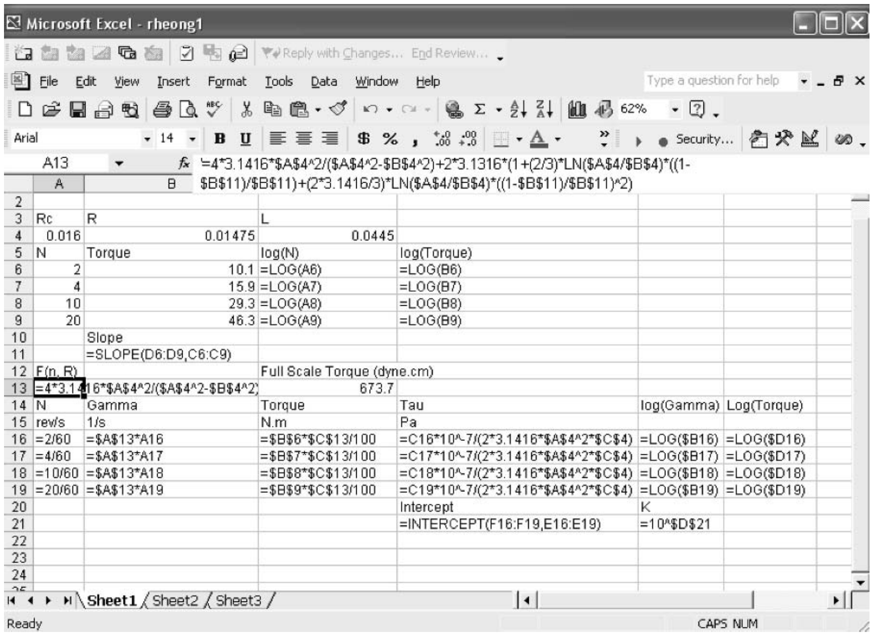
From the spreadsheet, (Figure 6.17), the slope of the regression of log(torque) versus log(RPM) is 0.66. Thus the value of n used to calculate the multiplying factor for N to obtain the shear rate from the rotational speed. The consistency index is the antilog of the intercept of a regression of log(τ) against log(γ). The intercept is -1.246 and the value of $K = 10^{-1.246} = 0.057\ \text{Pa} \cdot \text{s}^n$.

6.2.4.1 Wide Gap Rotational Viscometer

A wide gap rotational viscometer consists of a cylinder or a disk shaped bob on a spindle, which rotates in a pool of liquid. A torque transducer attached to the base of the rotating spindle measures the drag offered by the fluid to the rotation of the bob. Different size spindles may be used. Based on the spindle size and the rotational speed, the indicated torque can be converted to an apparent viscosity by using an appropriate conversion factor specific for the size of spindle and rotational speed. For a shear thinning fluid, the apparent viscosity increases as the rotational speed decreases. A log-log plot of the apparent viscosity against rotational speed will have a slope equivalent to $n - 1$. If one spindle is used at several rotational speeds, the unconverted torque reading is directly proportional to the shear stress, and the rpm is directly proportional to the shear rate. A log-log plot of torque reading against rpm will have a slope equivalent to the flow behavior index n.

Example 6.8. A Brookfield viscometer model RVF was used to evaluate the apparent viscosity of tomato catsup. One spindle (No. 4) gave readings within the measuring scale of the instrument at four rotational speeds. The viscometer constant was 7187 dyne ≅ cm full scale. The torque reading in per cent full scale at various rotational speeds in revolutions/min (rpm), respectively are as follows: (53.5, 2); (67, 4); (80.5,10); (97, 20).

(a)

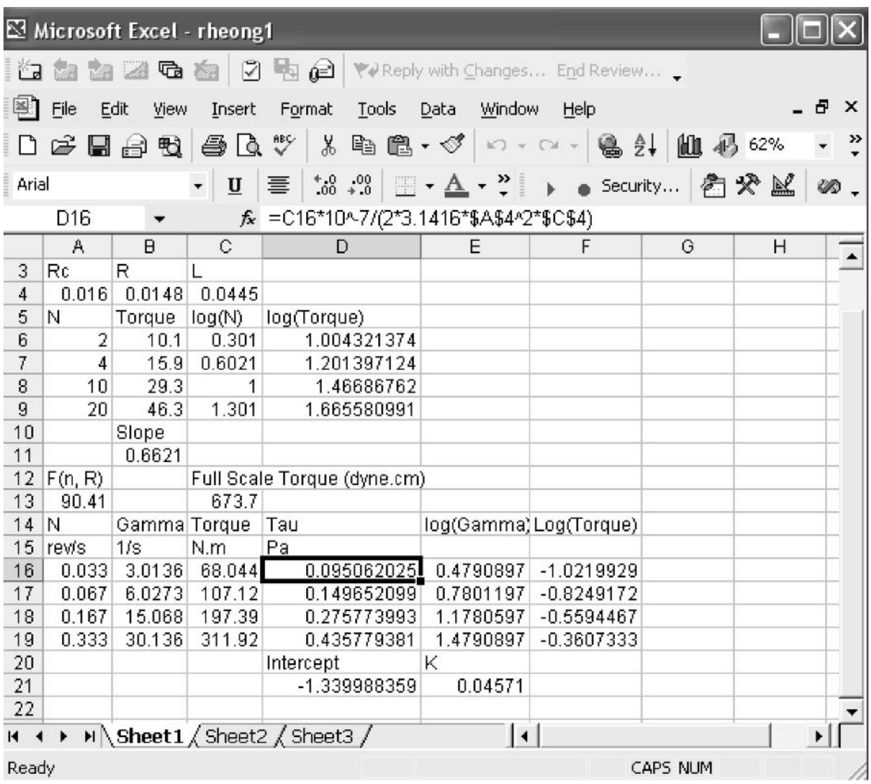


Microsoft Excel - rheong1

A13 f_x =4*3.1416*$A$4^2/($A$4^2-$B$4^2)+2*3.1316*(1+(2/3)*LN($A$4/$B$4)*((1-$B$11)/$B$11)+(2*3.1416/3)*LN($A$4/$B$4)*((1-$B$11)/$B$11)^2)

	A	B	C	D	E	F
3	Rc	R	L			
4	0.016	0.01475	0.0445			
5	N	Torque	log(N)	log(Torque)		
6	2	10.1	=LOG(A6)	=LOG(B6)		
7	4	15.9	=LOG(A7)	=LOG(B7)		
8	10	29.3	=LOG(A8)	=LOG(B8)		
9	20	46.3	=LOG(A9)	=LOG(B9)		
10		Slope				
11		=SLOPE(D6:D9,C6:C9)				
12	F(n, R)		Full Scale Torque (dyne.cm)			
13	=4*3.1416*$A$4^2/($A$4^2-$B$4^2)		673.7			
14	N	Gamma	Torque	Tau	log(Gamma)	Log(Torque)
15	rev/s	1/s	N.m	Pa		
16	=2/60	=$A$13*A16	=$B$6*$C$13/100	=C16*10^-7/(2*3.1416*$A$4^2*$C$4)	=LOG($B16)	=LOG($D16)
17	=4/60	=$A$13*A17	=$B$7*$C$13/100	=C17*10^-7/(2*3.1416*$A$4^2*$C$4)	=LOG($B17)	=LOG($D17)
18	=10/60	=$A$13*A18	=$B$8*$C$13/100	=C18*10^-7/(2*3.1416*$A$4^2*$C$4)	=LOG($B18)	=LOG($D18)
19	=20/60	=$A$13*A19	=$B$9*$C$13/100	=C19*10^-7/(2*3.1416*$A$4^2*$C$4)	=LOG($B19)	=LOG($D19)
20				Intercept	K	
21				=INTERCEPT(F16:F19,E16:E19)	=10^$D$21	

(b)



Microsoft Excel - rheong1

D16 f_x =C16*10^-7/(2*3.1416*$A$4^2*$C$4)

	A	B	C	D	E	F
3	Rc	R	L			
4	0.016	0.0148	0.0445			
5	N	Torque	log(N)	log(Torque)		
6	2	10.1	0.301	1.004321374		
7	4	15.9	0.6021	1.201397124		
8	10	29.3	1	1.46686762		
9	20	46.3	1.301	1.665580991		
10		Slope				
11		0.6621				
12	F(n, R)		Full Scale Torque (dyne.cm)			
13	90.41		673.7			
14	N	Gamma	Torque	Tau	log(Gamma)	Log(Torque)
15	rev/s	1/s	N.m	Pa		
16	0.033	3.0136	68.044	0.095062025	0.4790897	-1.0219929
17	0.067	6.0273	107.12	0.149652099	0.7801197	-0.8249172
18	0.167	15.068	197.39	0.275773993	1.1780597	-0.5594467
19	0.333	30.136	311.92	0.435779381	1.4790897	-0.3607333
20				Intercept	K	
21				-1.339988359	0.04571	

Figure 6.17 (a) Spreadsheet program in Microsoft Excel showing data entered and calculated values of shear stress, shear rate, flow behavior index, n, and consistency index, K. (b) Spreadsheet program in Microsoft Excel showing formulas in the cells used to calculate values in Fig. 6.17a.

Solution:

n is the slope of a log-log plot of torque against rpm.

Speed (rpm)	*Indicator Reading (% Full Scale)*	*Torque (dyne-cm)*
2	53.5	3845
4	67	48l5
10	80.5	5786
20	97	697l

The data plotted in Fig. 6.18 shows a slope of 0.25, which is the value of n.

6.2.4.2 Wide Gap Viscometer with Cylindrical Spindles

An important characteristic of a wide gap viscometer is that the fluid rotates in the proximate vicinity of the surface of the rotating spindle at the same velocity as the spindle (zero slip) and the fluid velocity

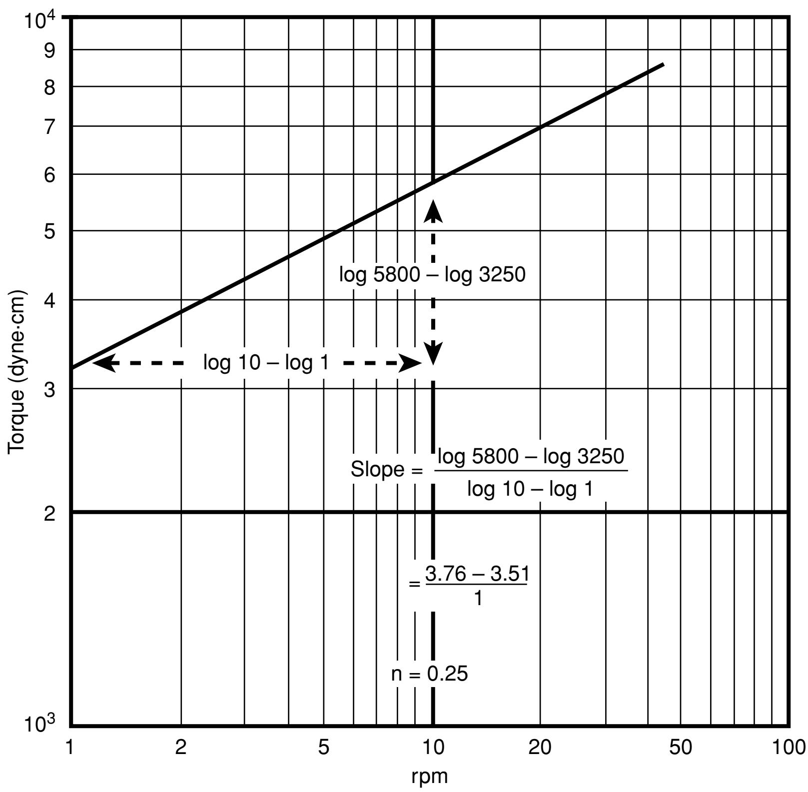


Figure 6.18 Log-log plot of torque against rotational speed to obtain the flow behavior index from the slope.

reaches zero at some point before the wall of the stationary outer cylinder. If these characteristics are met in a system, Kreiger and Maron (1952) derived an expression for the shear rate at the rotating inner cylinder wall as follows.

Let ω = angular velocity at a distance r from the center of the rotating spindle. $\gamma_w = (-dV/dr)$. Because $V = \omega$ r, then: $dV = r d\omega$

$$-\frac{dV}{dr} = r\,d\frac{\omega}{dr}$$

The shear stress "τ" at a point in the fluid a distance "r" from the center of the rotating spindle is:

$$\tau = \frac{\text{Torque}}{2\pi r^2 h} \quad r^2 = \frac{\text{Torque}}{2\pi\tau h}$$

Differentiating the expression for shear stress

$$d\tau = \frac{\text{Torque}}{2\pi h} - 2\frac{dr}{r^3}$$

Substituting r^2

$$d\tau = \frac{\text{Torque}}{2\pi h}\left(-2\frac{dr}{r}\right)\left(\frac{2\pi h}{\text{Torque}}\right)$$

$$d\tau = \frac{-2\tau\,dr}{r}$$

Solving for dr/r : $dr/r = (-1/2)d\tau/\tau$
Because $(-dV/dr) = F(\tau)$; then $r\,(d\omega/dr) = F(\tau)$, and $d\omega = (dr/r)F(\tau)$.
Substituting $(dr/r) = d\tau/2\tau$; $d\omega = (d\tau/2\tau)F(\tau)$ and $d\omega = (1/2)F(\tau)\,d\ln(\tau)$.
At the surface of the spindle, the shear stress is τ_w and the angular velocity is $2\pi N$. Thus:

$$d(2\pi N) = (1/2)\,F(\tau_w)\,d\ln(\tau_w).\ \text{Because } (-dV/dr)_w = F(\tau_w) \text{ Then } 2\pi dN = (1/2)(-dV/dr)\,d\ln(\tau_w)$$
$$\text{and: } -(dV/dr)_w = 4\pi dN/d\ln(\tau_w)$$

Because $d\ln(N) = dN/N$, then $dN = N\,d\ln(N)$

$$-(dV/dr)_w = 4\pi N/[d\ln(\tau_w)/d\ln(N)]$$

The denominator is the slope of the log-log plot of torque versus N and has the same value as the flow behavior index n.

$$\gamma_w = \frac{4\pi N}{n} \tag{6.31}$$

Brookfield Engineering Laboratories gives the following equation for shear rate at a point x distant from the center of the rotating cylindrical spindle in a wide gap viscometer. R = the radius of the spindle, and Rc = the radius of the cup.

$$\gamma = \frac{4\pi N Rc^2 R^2}{x^2(Rc^2 - R^2)} \tag{6.31a}$$

If Rc is much bigger than R, then $Rc^2/(Rc^2 - R^2)$ therefore, at the surface of the spindle where x = R, the shear rate is $\gamma = 4\pi N$. Thus Equations (6.30), (6.31) and (6.31a) are equivalent when n = 1.

To use Equation (6.31), a value of n is needed. The value of n is determined by plotting log(Torque) against log(N). The slope is the value of n. Knowing n, Equation (6.31) can then be used to calculate γ_w. τ_w is calculated using Equation (6.29). A log-log plot of τ_w against γ_w will have K for an intercept.

Example 6.9. The following data were obtained in the determination of the flow behavior of a fluid using a rotational viscometer. The viscometer spring constant is 7197 dyne ≅ cm full scale. The spindle has a diameter of 0.960 cm and a height of 4.66 cm.

N (rev/min)	*Torque (Fraction of full scale)*
2	0.155, 0.160, 0.157
4	0.213, 0.195, 0.204
10	0.315, 0.275, 0.204
20	0.375, 0.355, 0.365

Calculate the consistency and flow behavior indices for this fluid.

Solution:

Equation (6.29) will be used to calculate τ_w, and Equation (6.31) for γ_w.

It will be necessary to determine n before Equation (6.31) can be used. The slope of a log-log plot of torque against the rotational speed will be the value of n. This plot shown in Fig. 6.19 shows a slope

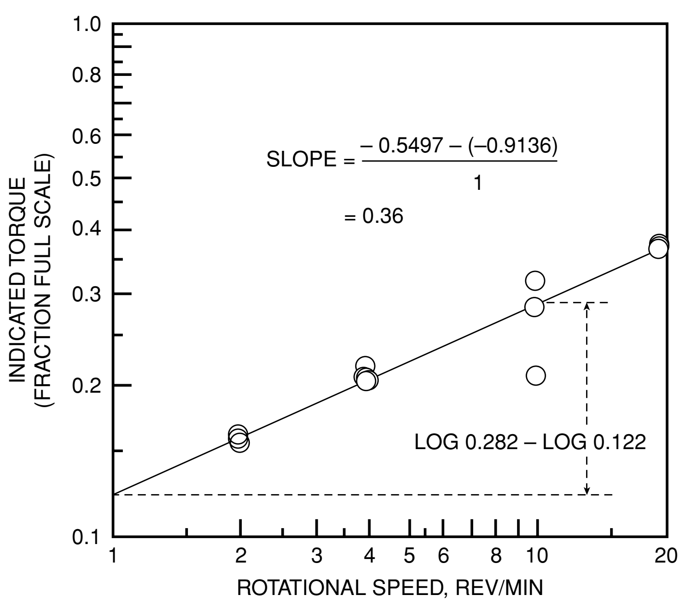


Figure 6.19 Log-log plot of indicated torque against the rotational speed to obtain the flow behavior index, n, from the slope.

of 0.36, which is the value of n. The multiplying factor to convert the rotational speed in rev/min to the shear rate at the wall of the rotating cylinder, γ_w , in s^{-1} units is: 4(3.1416)/[0.36(60)] = 0.5818. The multiplying factor to convert the indicated torque as a fraction of viscometer full scale torque to the shear stress at the rotating cylinder wall, τ_w, in Pascals is

$$7187(10^{-7})/[2(3.1416)(0.0096/2)^2(0.0466)] = 106.5367.$$

The values for τ_w and Q_w are

Shear stress (τ_w) (Pa)	*Shear rate (γ_w) (1/s)*
16.5, 17.0, 16.7	1.16
22.7, 20.8, 21.7	2.33
33.6, 29.3, 21.7	5.82
39.9, 37.8, 38.9	11.64

The plot of τ_w against γ_w shown in Fig. 6.20 shows that $\tau_w = 15.2$ Pa at $\gamma_w = 1$; thus, K = 15.2 Pa Xs^n.

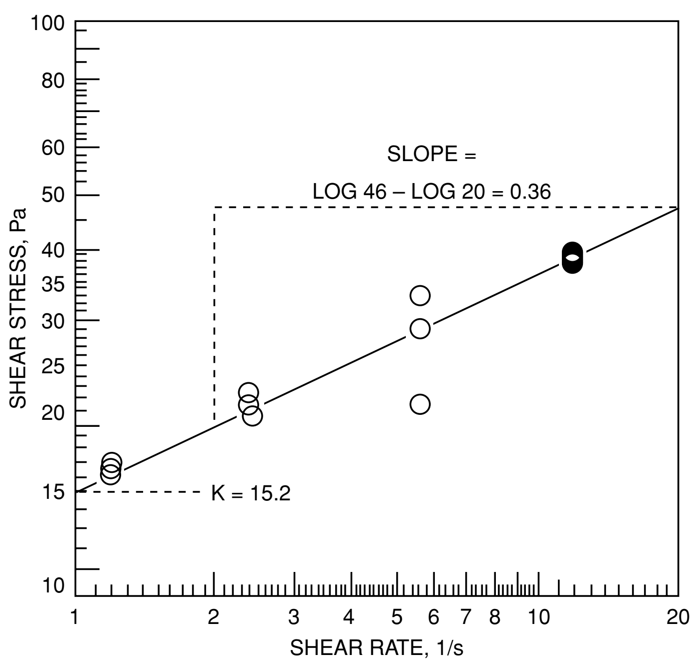


Figure 6.20 Log-log plot of shear stress against shear rate to obtain the consistency index, K, from the intercept.

6.3 CONTINUOUS VISCOSITY MONITORING AND CONTROL

In the formulation of foods where viscosity or consistency is a quality attribute, continuous monitoring of flow properties and automatic control of the feeding of ingredients is important. Two good examples of these processes are: blending of tomato paste with sugar, vinegar and flavoring in manufacturing of catsup; and blending of flour, water and flavoring ingredients to make batter for battered and breaded fried fish, poultry, or vegetables. These processes are rather simple because the level of only one ingredient in the formulation controls the consistency of the blend, and therefore will be very suitable for automatic control of flow properties. The principles for continuous viscosity monitoring are similar to those used in rheology. Some means must be used to divert a constant flow of fluid from the main pipeline to the measuring device. There are several configurations of viscosity measuring devices suitable for continuous monitoring in flowing systems. Three of the easiest to adapt on a system are described below.

6.3.1 Capillary Viscometer

A capillary viscometer may be used for continuous monitoring of consistency by tapping into the main piping, using a metering pump to deliver a constant flow through a level capillary, and returning the fluid to the main pipe downstream from the intake point. The metering pump must be a positive displacement pump to prevent flow fluctuations due to changes in the main pipeline pressure. A differential pressure transducer mounted at the entrance and exit of the capillary is used to measure the pressure drop. Poiseuille's equation (Eq. 6.11) is used to calculate the apparent viscosity from the pressure difference measured and the flow delivered by the metering pump.

6.3.2 Rotational Viscometer

Rotational viscometers could be used for continuous monitoring of viscosity in open vessels. Provision must be allowed for changes in fluid levels to ensure that the immersion depth of the spindle is appropriate. Mounting the instrument on a float device resting on the fluid surface, will ensure the same spindle immersion depth regardless of fluid level. Some models of rotational viscometers can remotely display the torque, others give a digital readout of the torque which can be easily read on the instrument.

For fluids flowing through pipes, problems may be encountered with high fluid velocities. Semi-continuous measurement may be made by periodically withdrawing fluid from the pipe through a sampling valve and depositing this sample into a cup where the viscosity is measured by a rotational viscometer. This type of measurement will have a longer response time than a truly in-line measurement.

6.3.3 Viscosity Sensitive Rotameter

This device is similar to a rotameter and operates on the principle that the drag offered by an obstruction in a field of flow is proportional to the viscosity and the rate of flow. A small float positioned within a vertical tapered cylinder will assume a particular position when fluid a fixed flow rate, and with a particular viscosity is flowing through the tube. This device needs a constant rate of flow to be effective, and usually, a metering pump draws out fluid from a tap on a pipe, delivers the fluid through the viscosity sensitive rotameter, and returns the fluid to the pipe through a downstream tap.

6.4 FLOW OF FALLING FILMS

Fluid films are formed when fluids flow over a vertical or inclined surface, or when fluid is withdrawn from a tank at a rapid rate. The differential equation for the shear stress and shear rate relationship for the fluid coupled with a differential force balance will allow a resolution of the thickness of the film as a function of fluid flow properties.

6.4.1 Films of Constant Thickness

When the rate of flow of a fluid over a vertical or inclined plane is constant, the film thickness is also constant. Figure 6.21 shows a fluid film flowing down a plane inclined at an angle α with the vertical plane. If the width of the plane is W, a force balance on a macroscopic section of the film of length L and of thickness x, measured from the film surface, is as follows:

Mass of the differential increment $= LW\rho x$
Force due to gravity on mass $= LW\rho gx$
Component of gravitational force along inclined plane $= LW\rho gx\cos(\alpha)$
Fluid resistance against the flow $= \tau WL$.

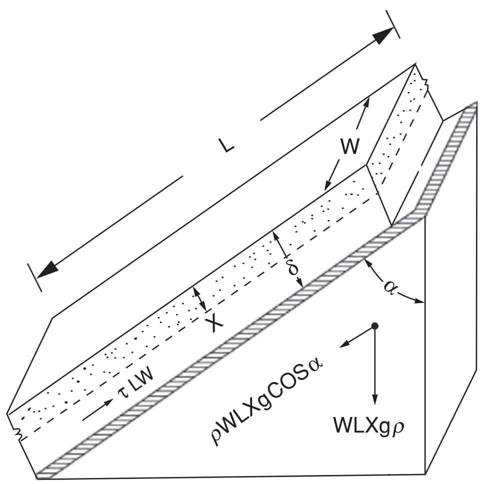


Figure 6.21 Diagram of a control element used in analysis of fluid film flowing down an inclined plane.

If the fluid follows the power law equation (Eq. 6.2), a force balance results in:

$$\mathrm{WLK}\left[-\frac{dV}{dr}\right]^n = \mathrm{LW}\rho\, g x \cos(\alpha)$$

Rearranging:

$$-\frac{dV}{dx} = \left[\frac{\rho\, g x \cos(\alpha)}{K}\right]^{1/n}$$

Integrating:

$$\int dV = -\left[\frac{\rho\, g \cos(\alpha)}{K}\right]^{1/n} \int [x]^{1/n}\, dx$$

The integrated expression is

$$V = -\left[\frac{\rho\, g \cos(\alpha)}{K}\right]^{1/n}\left[\frac{n}{n+1}\right][x]^{(n+1)/n} + C$$

The boundary condition, at $x = \delta$, $V = 0$, when substituted in the above expression will give the value of the integration constant, C. δ is the film thickness, and since x is measured from the surface of the film, the position $x = \delta$ is the surface of the incline along which the film is flowing. The final expression for the fluid velocity profile is

$$V = \left[\frac{\rho\, g \cos(\alpha)}{K}\right]^{1/n}\left[\frac{n}{n+1}\right][\delta]^{(n+1)/n}\left[1 - \left[\frac{x}{\delta}\right]^{(n+1)/n}\right]$$

The thickness of the film δ can be expressed in terms of an average velocity of flow, $\bar{V}$. The expression is derived as follows:

$$W\delta\bar{V} = \int_0^\delta VW\, dx = W\left[\frac{\rho\, g \cos(\alpha)}{K}\right]^{1/n}\left[\frac{n}{n+1}\right][\delta]^{(n+1)/n}\int_0^\delta \left[1 - \left[\frac{x}{\delta}\right]^{(n+1)/n}\right] dx$$

$$\bar{V} = \left[\frac{\rho\, g \cos(\alpha)}{K}\right]^{1/n}\left[\frac{n}{2n+1}\right][\delta]^{(n+1)/n} \tag{6.32}$$

The film thickness is

$$\delta = \left[\frac{\bar{V}(2n+1)K^{1/n}}{n[\rho\, g \cos(\alpha)]^{1/n}}\right]^{n/(n+1)} \tag{6.33}$$

Example 6.10. Applesauce at 80°C ($K = 9$ Pa$\cdot$ s^n; $n = 0.33$, $\rho = 1030$ kg/m^3 is to be de-aerated by allowing to flow as a film down the vertical walls of a tank 1.5 m in diameter. If the film thickness desired is 5 mm, calculate the mass rate of flow of applesauce to be fed into the de-aerator. At this film thickness, and 80°C, an exposure time of film to a vacuum of 25 in. Hg of 15 seconds is required to reduce dissolved oxygen content to a level needed for product shelf stability. Calculate the height of the tank needed for the film to flow over.

Solution:

Equation (6.32) will be used to calculate the average velocity of the film needed to give a film thickness of 5 mm. Since the side is vertical, $\alpha = 0$ and $\cos \alpha = 1$.

$$\bar{V} = \left[\frac{(1030)(9.8)}{9.0}\right]^{1/0.33}\left[\frac{(0.005)^{4.03}}{5.03}\right] = 0.1845\,\mathrm{m/s}$$

The mass rate of flow, $\bar{m}$, is

$$\bar{m} = \frac{0.1845\,\mathrm{m}}{\mathrm{s}}(\pi)[(0.75)^2 - (0.75 - .005)^2]\mathrm{m}^2\left(\frac{1030\,\mathrm{kg}}{\mathrm{m}^3}\right)$$
$$= 4.48\,\mathrm{kg/s}$$

The height of the vessel needed to provide the 15 seconds required exposure for de-aeration is

$$h = \frac{0.1845\,\mathrm{m}}{\mathrm{s}}(15\,\mathrm{s}) = 2.77\,\mathrm{m}$$

6.4.2 Time-Dependent Film Thickness

Problems of this type are encountered when fluids cling as a film on the sides of a storage vessel when the vessel is emptied (drainage), or when a solid is passed through a pool of fluid and emerges from the fluid coated with a fluid film (withdrawal). The latter process may be encountered when applying batter over a food product prior to breading and frying. The solution to problems of withdrawal and drainage is the same.

Figure 6.22 shows a fluid film with width W falling down a vertical surface. A control segment of the film having a thickness δ located a distance z from the top , and a length dz along the surface on which it flows, is shown. The solution will involve solving partial differential equations which involves a mass balance across the control volume.

If $\bar{V}$ is the average velocity of the fluid in the film at any point z distant from the top, the mass of fluid entering the control volume from the top is

$$M_i = \bar{V}\delta W\rho$$

The mass leaving the control volume from the lower part of the control volume is

$$M_e = \left[\bar{V}\delta + \frac{\partial}{\partial z}(\bar{V}\delta)dz\right]W\rho$$

The accumulation term due to the difference between what entered and left the control volume is

$$\frac{\partial \delta}{\partial z}\rho W dz$$

The mass balance across the control volume is

$$\bar{V}\delta W\rho = \left[\bar{V}\delta + \frac{\partial}{\partial z}(\bar{V}\delta)dz\right]\rho W + \frac{\partial \delta}{\partial z}\rho W\,dz$$

Simplification will produce the partial differential equation which describes the system.

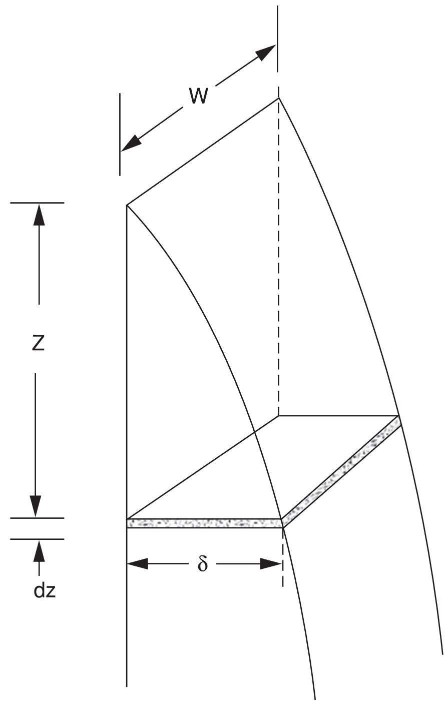


Figure 6.22 Diagram of a control element used in analysis of time dependent flow of fluid film down a surface.

$$\frac{\partial}{\partial z}(\bar{V}\delta) = -\frac{\partial \delta}{\partial t} \tag{6.34}$$

The solution to this partial differential equation will be the product of two functions, G(t) which is a function only of t, and F(z) which is a function only of z.

$$\delta = F(z)G(t) \tag{6.35}$$

Partial differentiation of Equation (6.35) gives:

$$\frac{\partial \delta}{\partial t} = F(z)\frac{\partial G(t)}{\partial t} \tag{6.36}$$

$$\frac{\partial \delta}{\partial z} = G(t)\frac{\partial F(z)}{\partial z} \tag{6.37}$$

Equation (6.32) represents $\bar{V}$ for a fluid film flowing down a vertical surface ($\cos\alpha = 1$). Partial differentiation of $\bar{V}\delta$ with respect to z gives:

$$\begin{aligned}\frac{\partial}{\partial z}(\bar{V}\delta) &= \frac{\partial}{\partial z}\left[\left[\frac{\rho g}{K}\right]^{1/n}\left[\frac{n}{2n+1}\right]\left[\delta\right]^{(n+1)/n}\delta\right] \\ &= \left[\frac{\rho g}{K}\right]^{1/n}[\delta]^{(n+1)/n}\frac{\partial \delta}{\partial z}\end{aligned} \tag{6.38}$$

Substituting Equation (6.34) for $M/Mz\ (\bar{V}\delta)$; Equation (6.36) for $M\delta/Mt$; and Equation (6.37) for $M\delta/Mz$ in Equation (6.38):

$$-F(z)\frac{\partial G(t)}{\partial t} = \left[\frac{\rho}{K}\right]^{1/n}[F(z)G(t)]^{(n+1)/n}G(t)\frac{FF(z)}{\partial z}$$

Each side of the partial differential equation contains only one variable, therefore the equation can be separated and each side integrated independently by equating it to a constant. Let the constant $= \lambda$.

The function with the time variable is obtained by integrating the left hand side of the equation, setting it equal to λ . The function with the z variable is obtained the same way, using the right side of the equation.

$$F(z) = \left[\lambda\left[\frac{K}{\rho g}\right]^{1/n}\left[\frac{n}{n+1}\right]z\right]^{n/(n+1)} + C_2$$

$$G(t) = \left[\lambda\left[\frac{n+1}{n}\right]t\right]^{-n/(n+1)} + C_1$$

The boundary conditions, $\delta = 0$ at $z = 0$, and $t = 4$, will be satisfied when the integration constants C_1 and C_2 are zero. When expressions for F(z) and G(t) are combined according to Equation (6.35), the constant λ cancels out. The final expression for the film thickness as a function of time and position is

$$\delta = \left[\left[\frac{K}{\rho g}\right]^{1/n}\left[\frac{z}{t}\right]\right]^{n/(n+!)} \tag{6.39}$$

Example 6.11. Calculate the thickness of batter that would be attached the sides of a brick-shaped food product, at the midpoint of its height, if the product is immersed in the batter, removed, and allowed to drain for 5 seconds before breading. The batter has a flow behavior index of 0.75, a density of 1003 kg/m^3, and a consistency index of 15 $Pa \cdot s^n$. The vertical side of the product is 2 cm high.

Solution:

$z = 0.001$ m; $t = 5$ s. Using Equation (6.38):

$$\delta = \left[\left[\frac{15}{1003(9.8)}\right]^{1/0.75}\left(\frac{0.001}{5}\right)\right]^{0.75/1.75}$$
$$= 0.000599 \text{ m}$$

Example 6.12. Applesauce is rapidly drained from a drum 75 cm in diameter and 120 cm high. At the end of 60 seconds after the fluid is drained from the drum, calculate the mass of applesauce adhering to the walls of the drum. The applesauce has a flow behavior index of 0.34, a consistency index of 11.6 $Pa \cdot s^n$, and a density of 1008 kg/m^3.

Solution:

Equation (6.39) will be used. However, the thickness of the film will vary with the height, therefore it will be necessary to integrate the quantity of fluid adhering at the different positions over the height of the vessel. At any given time, t, the increment volume of fluid dq, over an increment of height dz will be

$$dq = \pi[r^2 - (r - \delta)^2]\, dz$$

The total volume of film covering the height of the drum will be

$$q = \int_0^z \pi(2r\delta - \delta_2)\, dz$$

Substituting the expression for δ (Eq. 6.39) and integrating over z, holding time constant:

$$q = 2\pi r \left[\frac{2}{\rho g}\right]^{1/(n+1)} \left[\frac{1}{t}\right]^{n/(n+1)} \left[\frac{n+1}{2n+1}\right] [z]^{(2n+1)/(n+1)}$$
$$-\pi \left[\frac{K}{\rho}\right]^{2/(n+1)} \left[\frac{1}{t}\right]^{2n/(n+1)} \left[\frac{n+1}{3n+1}\right] [z]^{(3n+1)/(n+1)}$$

Substituting values of known quantities:

$$K/\rho g = 11.6/(1008)(9.8) = 0.001174$$
$$2/(n+1) = 1.515;\ 2n/(n+1) = 0.507;\ (n+1)/(3n+1) = 0.6633$$
$$1/(n+1) = 0.7463;\ n/(n+1) = 0.254;\ (n+1)/2n+1) = 0.7976$$
$$\begin{aligned} V &= 2\pi(0.375)(0.001174)^{0.7463}(1/60)^{0.254}(0.7976) \\ &= (1.2)^{1/0.7976} - \pi(0.001174)^{1.515}(1/60)^{0.507} \\ &= (0.6633)(1.2)^{1/0.6633} \\ &= 0.0054289 - 0.0000125101 = 0.005416\, m^3 \end{aligned}$$

The mass of applesauce left adhering to the walls of drum is

$$0.005416\, m^3 (1008) kg/m^3 = 5.46\, kg$$

6.4.3 Processes Dependent on Fluid Film Thicknesses

Fluid films are involved in processes that require rapid rates of mass and/or heat transfer. Examples are as follows.:

Ultra-high temperature sterilization of fluid milk and cream: One system (DaSi) rapidly heats a fluid film as it flows down multiple tubes in a superheated steam atmosphere. The steam and fluid are introduced in parallel. The fluid is pumped to a specially constructed distributor inside a cylindrical pressure vessel where it flows down the outside periphery of the vertically oriented tube in a thin film. Heat transfer into the thin film from the superheated steam with which it is in direct contact is very rapid resulting in very short residence time to heat the product to sterilization temperatures. The short heating time minimizes adverse flavor changes on the product. The fluid flow rate and the outside surface area of the tubes must be matched to obtain the necessary film thickness to permit the rapid heating.

Falling film evaporators: Fluid flows as a film down the surface of a heated tube(s) and discharges into a plenum where a vacuum allows liquid to vaporize by flash evaporation. To maintain efficiency of heating, fluid flow must be at a rate that would permit the desired thickness of fluid film to develop without flooding the whole inside cross-sectional area of the tube.

De-aeration of food fluids: Viscous fluids at elevated temperatures are exposed to a vacuum while flowing in a thin film to facilitate removal of dissolved oxygen. One system (FMC) sprays the fluid at the top of a tall column allowing the fluid to it the walls and flow down as a falling fluid film. The column dimensions needed to successfully de-aerate the product may be calculated as shown in the example in the previous section "Films of Constant Thickness."

Stripping of volatile flavor component from foods: A system developed by Flavourtech called a "Spinning Cone Column" consists of a rotating shaft to which multiple cones arranged at even intervals along the height of the shaft are attached. The cone faces upwards with the base radiating outward from the shaft and the apex of the cone is pierced by the rotating shaft to which the cone is fastened. Another stationary cone concentric with the rotating cone but open at the apex and attached at the periphery of the base to the wall of the cylindrical column, creates a channel to guide the fluid film as it travels from one cone element to the next. Volatile flavor compounds are stripped from the fluid using an inert gas flowing countercurrent to the fluid or by vapors generated by flash evaporation when the fluid is introduced into the column at a high temperature and a vacuum is maintained in the column. The top cone is a rotating cone and fluid entering the column is deposited at the apex of this cone. The fluid film flows upwards on the wall of the rotating cone due to centrifugal force of rotation and spills over the base periphery where it falls into the top of a stationary cone. Fluid film flows by gravity over the stationary cone toward the central shaft where it drops into the apex of another rotating cone. The process repeats between the alternating rotating and stationary cones. The system has been shown to be effective in stripping volatile flavors from an aqueous slurry of tea, recovery of volatile flavors from fruit and vegetable juices and purees, de-aeration of fruit juice and purees, removing alcohol from wine or beer, and in recovery of flavors from exhaust air streams such as onion dehydrator or fruit dehydrator. The mass transfer from a large area of thin fluid film makes this system very effective compared to other types of contractors used in extraction operations.

6.5 TRANSPORTATION OF FLUIDS

The basic equations characterizing flow and forces acting on flowing fluids are derived using the laws of conservation of mass, conservation of momentum, and the conservation of energy.

6.5.1 Momentum Balance

Momentum is the product of mass and velocity and has units of kg · m/s. When mass is expressed as a mass rate of flow, the product with velocity s the rate of flow of momentum and will have units of kg · m/s^2, the same units as force. When a velocity gradient exists in a flowing fluid, momentum transfer occurs across streamlines, and the rate of momentum transfer per unit area, d(mV/A)/dt, the momentum flux, has units of kg/m · s^2. The units of momentum flux is the same as that of stress or pressure. The pressure drop in fluids flowing through a pipe which is attributable to fluid resistance, is the result of a momentum flux between streamlines in a direction perpendicular to the direction of flow. The momentum balance may be expressed as:

$$\text{Rate of momentum flow in} + \sum F = \text{Rate of momentum flow out} + \text{Accumulation}$$

$\sum F$ is the sum of external forces acting on the system (e.g., atmospheric pressure, stress on the confining vessel) or forces exerted by restraints on a nozzle discharging fluid. Because momentum is a function of velocity, which is a vector quantity (i.e., it has both magnitude and direction), all the terms in the above equation are vector quantities. The form of the equation given above implies that a positive sign on EF indicates a force applied in a direction entering the system. When making a momentum balance, the component of the force or velocity acting in a single direction (e.g., x, y, or z component)is used in the equation. Thus, in three dimensional space, a momentum balance may be made for the x, y, and z directions.

The force balance used to derive the Poiseuille equation (Eq. 6.9) is an example of a momentum balance over a control volume, a cylindrical shell of fluid flowing within a cylindrical conduit. Equation (6.9) shows that the momentum flow (momentum flux multiplied by the surface area of the control volume) equals the net force (pressure drop multiplied by the crosssectional area of the control volume) due to the pressure acting on the control element.

When the control volume is the whole pipe (i.e., the wall of the pipe is the boundary surrounding the system under consideration), it will be possible to determine the forces acting on the pipe or its restraints. These forces are significant in instances where a fluid changes direction as in a bend, when velocity changes as in a converging pipe section, when a fluid discharges out of a nozzle, or when a fluid impacts a stationary surface.

Momentum flow (rate of change of momentum, $\bar{M}$) has the same units as force, and is calculated in terms of the mass rate of flow, $\bar{m}$, kg/s, the velocity, V, or the volumetric rate of flow, q, as follows:

$$\bar{M} = \bar{m}V = AV^2\rho = qV\rho$$

A is the area of the flow stream perpendicular to the direction of flow, and is the fluid density. The momentum flux (rate of change of momentum per unit area) is the quotient, $\bar{M}/A$. A force balance over a control volume is the same as a balance of momentum flow.

Example 6.13. Calculate the force acting on the restraints of a nozzle from which fluid is discharging to the atmosphere at the rate of 5 kg/s. The fluid has a density of 998 kg/m^3. It enters the nozzle at a pressure of 238.3 kPa. above atmospheric pressure. The nozzle has a diameter of 6 cm at the inlet and 2 cm at the discharge.

Solution:

Assume that the velocity of the fluid through the nozzle is uniform (i.e., there is no velocity variation across the pipe diameter). The pipe wall may be used as a system boundary for making the momentum balance, in order to include the force acting on the restraint. The momentum balance with subscripts 1 and 4 referring to the inlet and discharge of the nozzle respectively, is

$$\bar{m}V_1 + P_1A_1 + F_x = \bar{m}V_4 + P_4A_4$$

Atmospheric pressure (P_a) acts on the inlet to the nozzle entering the control volume, and this is also the pressure at the discharge. Because P_1 is pressure above atmospheric at the inlet, the momentum balance becomes:

$$\bar{m}V_1 + P_aA_1 + P_1A_1 + F_x = \bar{m}V_4 + P_aA_4$$

Solving for F_x:

$$F_x = \bar{m}(V_4 - V_1) - P_a(A_1 - A_4) - P_1A_1$$

Substituting known quantities and solving for V_1 and V_4:

$$A_1 = 0.002827\,m^2;\ A_4 = 0.001257$$

$$V_1 = \frac{5\,kg}{s}\frac{m^3}{998\,kg}\frac{1}{0.002827\,m^2} = 1.77\,m/s$$

$$V_4 = \frac{5\,kg}{s}\frac{m^3}{998\,kg}\frac{1}{0.001257\,m^2} = 3.99\,m/s$$

$$F_X = 5(3.99 - 1.77) - 438000(0.002827) - 101{,}300(0.002827 - 0.001257)$$
$$= 11.1 - 1238.2 - 159 = -1386\,N$$

The negative sign indicates that this force is acting in a direction opposite the momentum flow out of the system, thus the nozzle is being pushed away from the direction of flow.

6.5.2 The Continuity Principle

The principle of conservation of mass in fluid dynamics is referred to as the continuity principle. This principle is applied whenever there is a change of velocity, a change in diameter of the conduit, split flow, or a change in density of compressible fluids such as gases. The continuity principle for flow in one direction, expressed in equation form is as follows:

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 + A\frac{\partial}{\partial t}(\rho V) \tag{6.40}$$

If velocity is changing in all directions, a three-dimensional mass balance should be made. The above equation is a simplified form suitable for use in fluid transport systems where velocity is directed in one direction. At a steady state, the time derivative is zero. If the fluid is incompressible, the density is constant, and the continuity equation reduces to:

$$V_1 A_1 = V_2 A_2 \tag{6.41}$$

Example 6.14. A fluid having a density of 1005 kg/m3 is being drawn out of a storage tank 3.5 m in diameter through a tap on the side at the lowest point in the tank. The tap consists of a short length of 1.5 in. nominal sanitary pipe (ID = 0.03561 m) with a gate valve. If fluid flow out of the tap at the rate of 40 L/min, calculate the velocity of the fluid in the pipe and the velocity at which the fluid level recedes inside the tank.

Solution:

The mass rate of flow, $\bar{m}$ is constant. The mass rate of flow is the product of the volumetric rate of flow and the density. $\bar{m} = q\,\rho$

$$\bar{m} = \left[\frac{40\,L}{min}\cdot\frac{0.001\,m^3}{L}\cdot\frac{1\ min}{60\,s}\right]\cdot\left[\frac{1005\,kg}{m^3}\right] = 0.67\ kg/s$$

The velocity of fluid in the pipe is

$$V = \frac{q}{A} = \frac{\bar{m}}{\rho A}$$

$$V_{pipe} = \left[\frac{0.67\,\text{kg}}{\text{s}}\right] \cdot \frac{1\,\text{m}^3}{1005\,\text{kg}} \cdot \frac{1}{\pi(0.01781)^2\,\text{m}^2} = 0.669\,\text{m/s}$$

From the continuity principle:

$$A_{pipe}V_{pipe} = A_{Tank}V_{Tank}$$

$$V_{tank} = V_{pipe} \cdot \frac{A_{pipe}}{A_{tank}} = 0.669\frac{(0.03561)^2}{(3.5)^2} = 6.92 \times 10^{-5}\frac{\text{m}}{\text{s}}$$

6.6 FLUID FLOW REGIMES

Equation (6.10) indicates that a Newtonian fluid flowing inside a tube has a parabolic velocity profile. Fluid molecules in the center of the tube flows at the maximum velocity. The equation holds only when each molecule of the fluid remains in the same radial position as the fluid traverses the length of the tube. The path of any molecule follows a well-defined streamline, which can be readily shown by injecting a dye into the flow stream. When injected in a very fine stream, the dye will trace a straight line parallel to the direction of flow. This type of flow is called streamline or laminar flow. Figure 6.23 shows streamline flow that is observed when a dye is continuously injected into a flow stream (A), and the development of the parabolic velocity profile (D) that can be observed when a viscous dye solution is injected into a tube filled with a very viscous fluid at rest (B) and flow is allowed to develop slowly (C).

As rate of flow increases, molecular collisions occur at a more frequent rate and cross-overs of molecules across streamlines occur. Eddy currents develop in the flow stream. This condition of flow is turbulent and the velocity profile predicted by Equation (6.10) no longer holds. Figure 6.24 shows what can be observed when a dye is continuously injected into a fluid in turbulent flow. Swirling and curling of the dye can be observed. A certain amount of mixing also occurs. Thus, a particle originally introduced at the center of the tube may traverse back and forth between the wall and the center as the fluid travels the length of the tube. The velocity profile in turbulent flow is flat compared to the parabolic profile in laminar flow. The ratio of average to maximum velocity in laminar flow is 0.5 as opposed to an average ratio of approximately 0.8 in fully developed turbulent flow.

6.6.1 The Reynolds Number

The Reynolds number is a dimensionless quantity that can be used as an index of laminar or turbulent flow. The Reynolds number, represented by Re, is a function of the tube diameter, D, the average velocity $\bar{V}$, the density of the fluid ρ, and the viscosity μ.

$$Re = \frac{D\bar{V}\rho}{\mu} \tag{6.42}$$

For values of the Reynolds number below 2100, flow is laminar and the pressure drop per unit length of pipe can be determined using Equation (6.11) for a Newtonian fluid and Equation (6.5) for power law fluids. For Reynolds numbers above 2100, flow is turbulent and pressure drops are obtained using an empirically derived friction factor chart.

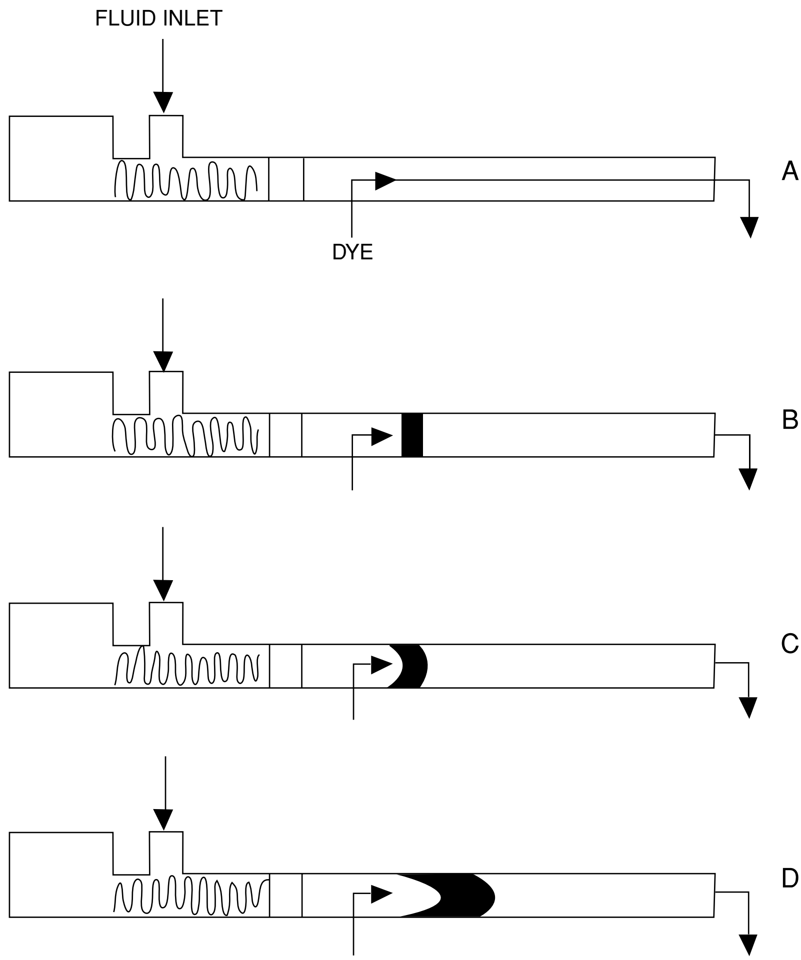


Figure 6.23 Development of laminar velocity profile during flow of a viscous fluid through a pipe.

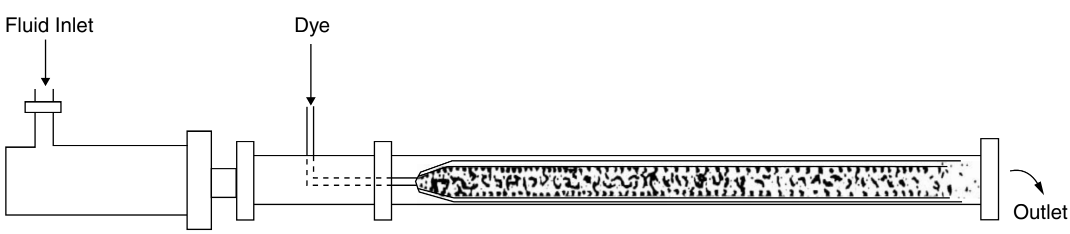


Figure 6.24 Path traced by a dye injected into a fluid flowing through a tube in turbulent flow.

Table 6.2 Pipe and Heat Exchanger Tube Dimensions (Numbers in Parentheses Represent the Dimension in Meters).

	Steel Pipe (Sch. 40)		*Sanitary Pipe*		*Heat Exchanger Tube (18 Ga)*	
Nominal size(in.)	*ID in./(m)*	*OD in./(m)*	*ID in./(m)*	*OD in./(m)*	*ID in./(m)*	*OD in./(m)*
0.5	0.622 (0.01579)	0.840 (0.02134)	–	–	0.402 (0.01021)	0.50 (0.0217)
0.75	0.824 (0.02093)	1.050 (0.02667)	–	–	0.652 (0.01656)	0.75 (0.01905)
1	1.049 (0.02664)	1.315 (0.03340)	0.902 (0.02291)	1.00 (0.0254)	0.902 (0.02291)	1.00 (0.0254)
1.5	1.610 (0.04089)	1.900 (0.04826)	1.402 (0.03561)	1.50 (0.0381)	1.402 (0.03561)	1.50 (0.0381)
2.0	2.067 (0.05250)	2.375 (0.06033)	1.870 (0.04749)	2.00 (0.0508)	–	–
2.5	2.469 (0.06271)	2.875 (0.07302)	2.370 (0.06019)	2.5 (0.0635)	–	–
3.0	3.068 (0.07793)	3.500 (0.08890)	2.870 (0.07289)	3.0 (0.0762)	–	–
4.0	4.026 (0.10226)	4.500 (0.11430)	3.834 (0.09739)	4.0 (0.1016)	–	–

ID = inside diameter.
OD = outside diameter.

6.6.2 Pipes and Tubes

Tubes are thin walled cylindrical conduits whose nominal sizes are based on the outside diameter. Pipes are thicker walled than tubes and the nominal size is based on the inside diameter. The term "sanitary pipe" used in the food industry is used for stainless steel tubing where the nominal designation is based on the outside diameter. Table 6.2 shows dimensions for steel and sanitary pipes, and heat exchanger tubes.

6.6.3 Frictional Resistance to Flow of Newtonian Fluids

Flow through a tubular conduit is accompanied by a drop in pressure. The drop in pressure is equivalent to the stress that must be applied on the fluid to induce flow. This stress can be considered as a frictional resistance to flow. Working against this stress requires the application of power in transporting fluids. An expression for the pressure drop as a function of the fluid properties and the dimension of the conduit would be useful in predicting power requirements for inducing flow over a given distance or for determining the maximum distance a fluid will flow with the given pressure differential between two points. For Newtonian fluids in laminar flow, the Poiseuille equation (Eq. 6.11) may be expressed in terms of the inside diameter (D) of a conduit, as follows:

$$\frac{\Delta P}{L} = \frac{32\bar{V}\mu}{D^2} \tag{6.43}$$

Equation (6.43) can be expressed in terms of the Reynolds number.

$$\frac{\Delta P}{L} = \frac{32\bar{V}\mu}{D^2} \cdot \frac{(D\bar{V}\rho)/\mu}{Re}$$

$$\frac{\Delta P}{L} = \frac{2\left(\frac{16}{Re}\right)(\bar{V})^2(\rho)}{D} \tag{6.44}$$

Equation (6.44) is similar to the Fanning equation derived by dimensional analysis where the friction factor f is 16/Re for laminar flow. The Fanning equation, which is applicable in both laminar and turbulent flow, is

$$\frac{\Delta P}{\rho} = \frac{2f(\bar{V})^2 L}{D} \tag{6.45}$$

For turbulent flow, the Fanning friction factor f may be determined for a given Reynolds number and a given roughness factor from a friction factor chart as shown in Fig. 6.25. For smooth tubes, a plot of log f obtained from the lowest curve in Fig. 6.25 against the log of Reynolds number would give the following relationships between f and Re:

$$f = 0.048\,(Re)^{-0.20} \quad 10^4 < Re < 10^6 \tag{6.46}$$

$$f = 0.193\,(Re)^{-0.35} \quad 3 \times 10^3 < Re < 10^4 \tag{6.47}$$

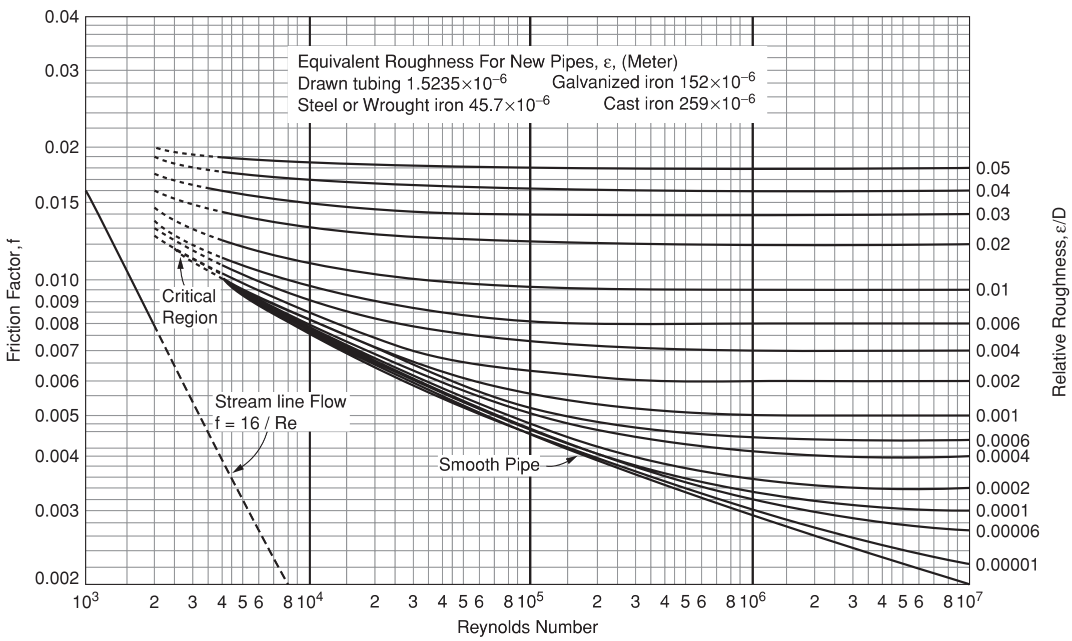


Figure 6.25 The Moody diagram for the Fanning friction factor. (Based on Moody, L. F. Friction factors for pipe flow. Trans. ASME. 66:671, 1944.)

Example 6.15. What pressure must be generated at the discharge of a pump that delivers 100 L/min of a fluid having a specific gravity of 1.02 and a viscosity of 100 centipoises? The fluid flows through a 1.5 in. (nominal) sanitary pipe, 50 m long. The pipe is straight and level, and the discharge end of the pipe is at atmospheric pressure.

Solution:

The Reynolds number is determined to determine if flow is laminar or turbulent.

$$\mathrm{Re} = \frac{D\bar{V}\rho}{\mu}$$

Substituting units of terms in the equation for Re:

$$\mathrm{Re} = \frac{D(\mathrm{m})\bar{V}(\mathrm{m/s})\rho(\mathrm{kg/m^3})}{\mu(\mathrm{Pa \cdot s})}$$

[Note that the Reynolds number will be dimensionless if the base units for Pa · s, kg/(m s), is substituted in the dimensional equation.]

Converting values of variables into SI units:

$$\rho = 1.02 \cdot \frac{1000\,\mathrm{kg}}{\mathrm{m^3}} = 1020\,\mathrm{kg/m^3}$$

$$D = 1.402\,\mathrm{in.} \cdot \frac{0.0254\,\mathrm{m}}{\mathrm{in.}} = 0.0356\,\mathrm{m}$$

$$\bar{V} = \frac{q}{\mathrm{area}} = \frac{0.00167\ \mathrm{m^3}}{\mathrm{s}} \cdot \frac{1}{\pi/4(0.0356)^2\ \mathrm{m^2}} = 1.677\ \mathrm{m/s}$$

$$q = 100\frac{1}{\mathrm{min}} \cdot \frac{0.001\,\mathrm{m^3}}{\mathrm{L}} \cdot \frac{1\,\mathrm{min}}{60\,\mathrm{s}} = 0.00167\,\mathrm{m^3/s}$$

$$\mu = (100\ \mathrm{cP} \cdot \frac{0.001\ \mathrm{Pa \cdot s}}{\mathrm{cP}} = 0.1\ \mathrm{Pa \cdot s}$$

Calculating the Reynolds number:

$$\mathrm{Re} = \frac{D\bar{V}\rho}{\mu} = \frac{(0.0356)(1.677)(1020)}{0.1} = 609$$

Flow is laminar. Using Equation (6.44):

$$\Delta P = \frac{2(16/\mathrm{Re})(\bar{V})^2(\rho)(L)}{D} = \frac{2(16)(1.677)^2(1020)(50)}{609(0.0356)}$$
$$= 211{,}699\ \mathrm{Pa\ gauge}$$

Because the pressure at the end of the pipe is 0 gauge pressure (atmospheric pressure), the pressure at the pump discharge will be

$$P = 211{,}699\ \mathrm{Pa\ gauge}$$

Example 6.16. Milk with a viscosity of 2 centipoises and a specific gravity of 1.01 is being pumped through a 1 in. (nominal) sanitary pipe at the rate of 3 gallons per minute. Calculate the pressure drop in lb_f/in^2 per foot length of level straight pipe.

Solution:

Use SI units and Equations (6.40) or (6.41) to calculate ΔP and convert to lb_f/in^2. From Table 6.2, a 1 in (nominal) sanitary pipe will have an inside diameter of 0.02291 m.

$$q = 3\frac{\text{gal}}{\text{min}} \cdot \frac{0.00378541\,\text{m}^3}{\text{gal}} \cdot \frac{1\,\text{min}}{60\,\text{s}} = 0.00018927\,\text{m}^3/\text{s}$$

$$\bar{V} = \frac{q}{A} = 0.00018927\frac{\text{m}^3}{\text{s}} \cdot \frac{1}{\pi/4(0.02291)^2\,\text{m}^3} = 0.459\,\text{m/s}$$

$$\rho = 1.01 \cdot \frac{1000\,\text{kg}}{\text{m}^3} = 1010\,\text{kg/m}^3$$

$$\text{Re} = \frac{D\bar{V}\rho}{\mu} = \frac{0.02291(0.459)(1010)}{0.002} = 5310$$

Because Re > 2100 flow is turbulent, use Equation (6.45) for calculating ΔP and Equation (6.47) for f at Re < 10^4. Sanitary pipe is considered a smooth pipe.

$$f = 0.193(\text{Re})^{-0.35} = 0.193(5310)^{-0.35} = 0.0095$$

f = 0.0095 can also be obtained from Fig. 6.25. The pressure drop per unit length of pipe may be calculated as $\Delta P/L$. Because the units used in the calculations are in SI, the result will be Pa/m. This can then be converted to the desired units of (lb $f_f/in.^2$)/ft. It may also be solved by substituting a value for L in Equation (6.45).

$$L = 1\text{ft}(0.3048\text{m/ft}) = 0.3048\text{m}.$$

$$\Delta P = \frac{2f\bar{V}^2 L\rho}{D} = \frac{2(0.0095)(0.459)^2(0.3048)(1010)}{0.02291}$$

$$= 53.79\,\text{Pa}$$

$$\Delta P = 53.79\ \text{Pa} \cdot \frac{1\ \text{lb}_f/\text{in}^2}{6894.757\ \text{Pa}} = 0.0078\frac{\text{lb}_f/\text{in}^2}{\text{ft of pipe}}$$

Example 6.17. Calculate the pressure drop for water flowing at the rate of 10 gal./min through 100 m of level straight wrought iron pipe having an inside diameter of 0.3579 m. Use a density for water of 62.4 lb/ft^3 and a viscosity of 0.98 centipoises.

Solution:

This problem is different from Examples 6.15 and 6.16 in that the pipe is not smooth and therefore Equations (6.46) or (6.47) cannot be used for calculating f. After calculating the Reynolds number, f can be determined from a Moody diagram (Fig. 6.25) if flow is turbulent, and ΔP can be calculated using Equation (6.45).

From Table 6.2: D = 0.03579 m

$$q = \frac{10\,\text{gal}}{\text{min}} \cdot \frac{0.00378541\,\text{m}^3}{\text{gal}} \cdot \frac{1\,\text{min}}{60\,\text{s}} = 0.0006309\,\text{m}^3/\text{s}$$

$$\text{Re} = \frac{D\bar{V}\rho}{\mu} = \frac{0.03579(0.6274)(999.7)}{0.00098} = 22{,}906$$

$$\rho = 62.4\frac{\text{lb}_\text{m}}{\text{ft}^3} \cdot \frac{(3.281)^3\text{ft}^3}{\text{m}^3} \cdot \frac{0.45359\,\text{kg}}{\text{lb}_\text{m}} = 999.7\,\text{kg/m}^3$$

$$\bar{V} = \frac{q}{A} = \frac{0.0006309\,\text{m}^3/\text{s}}{(\pi/4)(0.03579)^2(\text{m}^2)} = 0.6274\,\text{m/s}$$

$$\mu = 0.98\,\text{cP} \cdot \frac{0.001\,\text{Pa}\cdot\text{s}}{\text{cP}} = 0.00098\,\text{Pa}\cdot\text{s}$$

To use Fig. 6.25, first determine ε/D. From Fig. 6.24, $\varepsilon = 0.0000457$ m. $\varepsilon/D = 0.0000457/0.03579 = 0.00128$. For $\text{Re} = 2.29 \times 10^4$, and $\varepsilon/D = 0.00128$, $f = 0.0069$.

$$\Delta P = \frac{2(0.0069)(0.6274)^2(100)(999.7)}{0.03579} = 15.17\,\text{kPa}$$

6.6.4 Frictional Resistance to Flow of Non-Newtonian Fluids

For fluids whose flow behavior can be expressed by Equation (6.2) (pseudoplastic or power law fluids), the Rabinowitsch-Mooney equation (Eq. 6.18) for the shear rate at the wall was derived in the section "Velocity Profile and Shear Rate for a Power Law Fluid." Substituting the shear stress at the wall (Eq. 6.22) for τ_w and the shear rate at the wall, γ_w (Eq. 6.18), in Equation (6.2):

$$\frac{\Delta PR}{2L} = K\left[\frac{\bar{V}}{R} \cdot \frac{3n+1}{n}\right]^n$$

$$\Delta P = \frac{2LK(\bar{V})^n}{(R)^{n+1}}\left[\frac{3n+1}{n}\right]^n \tag{6.48}$$

Equation (6.44) for the pressure drop of a fluid in laminar flow through a tubular conduit may be written in terms of the radius of the tube and the Reynolds number as follows:

$$\Delta P = \frac{2(16/\text{Re})(\bar{V})^2\rho L}{2R} \tag{6.49}$$

Equation (6.49) is simply another form of Equation (6.44). These equations are very general and can be used for any fluid. In laminar flow, the friction factor, expressed as 16/Re, may be used as a means of relating the pressure drop of a non-Newtonian fluid to a dimensionless quantity, Re. Thus, Re for a non-Newtonian fluid can be determined from the flow behavior and consistency indices. Equating

Equations (6.48) and (6.49):

$$\frac{2LK(\bar{V})^n}{(R)^{n+1}}\left[\frac{3n+1}{n}\right]^n = \frac{2(16/Re)(\bar{V})^2 L\rho}{2R}$$

$$Re = \frac{8(\bar{V})^{2-n}(R)^n\rho}{K\left[\frac{3n+1}{n}\right]^n} \tag{6.50}$$

Equation (6.50) is a general expression for the Reynolds number that applies for both Newtonian ($n = 1$) and non-Newtonian liquids ($n \neq 1$). If $n = 1$, Equation (6.50) is exactly the same as Equation (6.42), when $K = \mu$. Pressure drops for power law non-Newtonian fluids flowing through tubes can be calculated using Equation (6.49) using the Reynolds number calculated using Equation (6.50), if flow is laminar. If flow is turbulent, the Reynolds number can be calculated using Equation (6.50) and the pressure drop calculated using Equation (6.45) with the friction factor Af A determined from this Reynolds number using Equations (6.46) or (6.47). The Moody diagram (Fig. 6.25) can also be used to determine "f" from the Reynolds number.

Example 6.18. Tube viscometry of a sample of tomato catsup shows that flow behavior follows the power law equation with $K = 125$ dyne As/cm^2 and $n = 0.45$. Calculate the pressure drop per meter length of level pipe if this fluid is pumped through a 1-in. (nominal) sanitary pipe at the rate of 5 gallons (US)/min. The catsup has a density of 1.13 g/cm^3.

Solution:

Convert given data to SI units. $n = 0.45$ (dimensionless).

$$K = 125\frac{\text{dyne s}^n}{\text{cm}^2}\cdot\frac{1\times10^{-5}\,\text{N}}{\text{dyne}}\cdot\frac{(100)^2\,\text{cm}^2}{\text{m}^2} = 12.5\,\text{Pa}\cdot\text{s}^n$$

From Table 6.2, $D = 0.02291$ m; $R = 0.01146$ m

$$q = \frac{5\,\text{gal}}{\text{min}} - \frac{0.00378541\text{m}^3}{\text{gal}} - \frac{1\,\text{min}}{60\,\text{s}} = 0.00031545\,\text{m}^3/\text{s}$$

$$\bar{V} = \frac{q}{A} = \frac{0.00031545\ \text{m}^3/\text{s}}{(\pi/4)(0.02291)^2\,\text{m}^2} = 0.7652\ \text{m/s}$$

$$\rho = 1.13\frac{\text{g}}{\text{cm}^3}\frac{(100)^3\ \text{cm}^3}{\text{m}^3}\frac{1\,\text{kg}}{1000\,\text{g}} = 1130\,\text{kg/m}^3$$

$$L = 1\,\text{m}$$

The Reynolds number is calculated using Equation (6.50).

$$Re = \frac{8(0.7652)^{2-0.45}(0.01146)^{0.45}(1130)}{12.5\left[\frac{3(.45)+1}{0.45}\right]^{0.45}} = 30.39$$

Flow is laminar and pressure drop is calculated using Equation (6.45).

$$\Delta P = \frac{2\left(\dfrac{16}{30.39}\right)(0.7652^2)(1130)(1)}{0.02291} = 30.41\ \text{kPa}$$

Example 6.19. Peach puree having a solids content of 11.9% has a consistency index K of 72 dyne As^n/cm^2 and a flow behavior index, n = 0.35. If this fluid is pumped through a 1-in. (nominal) sanitary pipe at 50 gallons (US)/min, calculate the pressure drop per meter of level straight pipe. Peach puree has a density of 1.07 g/cm^3.

Solution:

D = 0.02291 (from Table 6.2); R = 0.01146 m; n = 0.35

$$K = 72\ \frac{\text{dyne s}^n}{\text{cm}^2} \cdot \frac{(10)^{-5}}{\text{dyne}} \cdot \frac{(100)^2\ \text{cm}^2}{\text{m}^2} = 7.2\ \text{Pa} \cdot \text{s}^n$$

$$\rho = 1070\ \text{kg/m}^3$$

$$q = 50\frac{\text{gal}}{\text{min}}\ \frac{0.00378541\ \text{m}^3}{\text{gal}}\ \frac{1\ \text{min}}{60\ \text{s}} = 0.0031545\ \text{m}^3/\text{s}$$

$$\bar{V} = \frac{q}{A} = \frac{0.00378541\ \text{m}^3}{(\pi/4)(0.02291)^2} = 7.6523\ \text{m/s}$$

Using Equation (6.50):

$$Re = \frac{8(7.6523)^{2-0.35}(0.01146)^{0.35}(1070)}{7.2\left[\dfrac{3(0.35)+1}{0.35}\right]^{0.35}} = 3850$$

Flow is turbulent. The sanitary pipe is a smooth pipe so Equation (6.47) will be used to calculate f.

$$f = 0.193(Re)^{-0.35} = 0.193(3850)^{-0.35} = 0.010731$$

P can now be calculated using equation 42. L = 1 m.

$$\Delta P = \frac{2(0.01073)(7.6523)^2(1)(1070)}{0.02291} = 58.691\ \text{kPa/m}$$

6.6.5 Frictional Resistance Offered by Pipe Fittings to Fluid Flow

The resistance of pipe fittings to flow can be evaluated in terms of an equivalent length of straight pipe. Each type of pipe fitting has its specific flow resistance expressed as a ratio of equivalent length of straight pipe (L=) over its diameter. Table 6.3 lists the specific resistance of various pipe fittings. The equivalent length of a fitting, which is the product of L=/D obtained from Table 6.3 and the pipe diameter, is added to the length of straight pipe within the piping system to determine the total drop pressure drop across the system.

Table 6.3 Specific Resistance of Various Pipe Fittings Expressed as an Equivalent Length of Straight Pipe to Pipe Diameter Ratio.

Fitting	*L′/D* (*dimensionless*)
90° elbow std.	35
45°elbow std.	15
Tee (Used as a coupling), branch plugged	20
Tee (used as an elbow) entering the branch	70
Tee (used as an elbow) entering the tee run	60
Tee branching flow	45
Gate valve, fully open	10
Globe valve, fully open	290
Diaphragm valve, fully open	105
Couplings and unions	Negligible

Example 6.20. Calculate the pressure drop due to fluid friction across 50 m of a 1-in. (nominal) sanitary pipe that includes five 90-degree Eelbows in the piping system. Tomato catsup with properties described in Example 6.18 in the section "Frictional Resistance to Flow for non-Newtonian Fluids" flows through the pipe at the rate of 5 gallons (US)/min.

Solution:

In Example 6.18 of the section referred to above, the tomato catsup pumped under these conditions exhibited a pressure drop of 30.41 kPa/m of level pipe. The equivalent length of the five elbows is determined as follows:

L=/D from Table 6.3 for a 90-degree elbow is 35. The diameter of the pipe is 0.02291 m.
Total equivalent length of straight pipe = 50 m + 5 (35)(0.02291) = 54 m.
$\Delta P = (30.41\,\text{kPa/m})(54\text{m}) = 1642\,\text{kPa}$.

6.7 MECHANICAL ENERGY BALANCE: THE BERNOULLI EQUATION

When fluids are transferred from one point to another, a piping system is used. Because fluids exhibit a resistance to flow, an energy loss occurs as the fluid travels downstream along the pipe. If the initial energy level is higher than the energy at any point downstream, the fluid will flow spontaneously. On the other hand, if the energy change needed to take the fluid to a desired point downstream exceeds the initial energy level, energy must be applied to propel the fluid through the system. This energy is an input provided by a pump.

An energy balance across a piping system is similar to the energy balance made in Chapter 5. Table 6.4 lists the energy terms involved in fluid flow, their units, and the formulas for calculating them. All energy entering the system including the energy input must equal that leaving the system and the energy loss due to fluid friction. The boundaries of the system must be carefully defined to identify the input and exit energies. As in previous problems in material and energy balances, the boundaries

Table 6.4 Energy Terms Involved in the Mechanical Energy Balance for fluid Flow in a Piping System, the Formulas for Calculating Them, and Their Units.

Energy Term	*Formula*	*Dimensional Expression*	*Formula (basis: 1 kg)*	*Unit*
Potential energy Pressure	$m\left(\frac{P}{\rho}\right)$	$\frac{kg(N \cdot m^{-2})}{kgm^{-3}}$	$\frac{P}{\rho}$	joule/kg
Elevation	mgh	$kg(m \cdot s^{-2})(m)$	gh	joule/kg
Kinetic energy	$\frac{1}{2}mV^2$	$kg(m \cdot s^{-1})^2$	$\frac{V^2}{2}$	joule/kg
Work input (from pump)	W		W	joule/kg
Frictional resistance	$\frac{m\Delta P_f}{\rho}$	$\frac{kg(N \cdot m^{-2})}{kg \cdot m^{-3}}$	$\frac{\Delta P_f}{\rho}$	joule/kg

may be moved around to isolate the unknown quantities being asked in the problem. The mechanical energies involved are primarily kinetic and potential energy and mechanical energy at the pump.

The potential and kinetic energy could have finite values on both the source and discharge points of a system. Work input should appear on the side of the equation opposite the frictional resistance term. A balance of the intake and exit energies in any given system including the work input and resistance terms based on a unit mass of fluid is

$$\frac{P_1}{\rho} + gh_1 + \frac{{V_1}^2}{2} + W_s = \frac{P_2}{\rho} + gh_2 + \frac{{V_2}^2}{2} + \frac{\Delta P_f}{\rho} \tag{6.51}$$

Equation (6.51) is the Bernoulli equation.

Example 6.21. A pump is used to draw tomato catsup from the bottom of a de-aerator. The fluid level in the de-aerator is 10 m above the level of the pump. The de-aerator is being operated at a vacuum of 0.5 kg_f/cm^2 (kilogram force/cm^2). The pipe connecting the pump to the de-aerator is a 2.5-in. (nominal) stainless steel sanitary pipe, 8 m long with one 90-degree elbow. The catsup as a density of 1130 kg/m^3, a consistency index K of 10.5 $Pa \cdot s^n$, and a flow behavior index n of 0.45 (dimensionless). If the rate of flow is 40 L/min, calculate the pressure at the intake side of the pump to induce the required rate of flow.

Solution:

Figure 6.26 is a diagram of the system. The problem asks for the pressure at a point in the system before work is applied by the pump to the fluid. Therefore, the term W_s in Equation (6.51) is zero. The following must be done before quantities can be substituted into Equation (6.51).
Assume atmospheric pressure is 101 kPa. P_1 must be converted to absolute pressure. The volumetric rate of flow must be converted to velocity ($\underline{V}$). The frictional resistance to flow ($\Delta P_f/\rho$ must be calculated.

$$\text{Vacuum} = 0.5\ \frac{kg_f}{cm^2} \cdot \frac{0.8\ m \cdot kg}{kg_f \cdot s^2} \cdot \frac{(100)^2 cm^2}{m^2} = 49\ kPa$$

$$P_1 = \text{Atmospheric pressure} - \text{vacuum} = 101 - 49 = 52\ kPa.$$

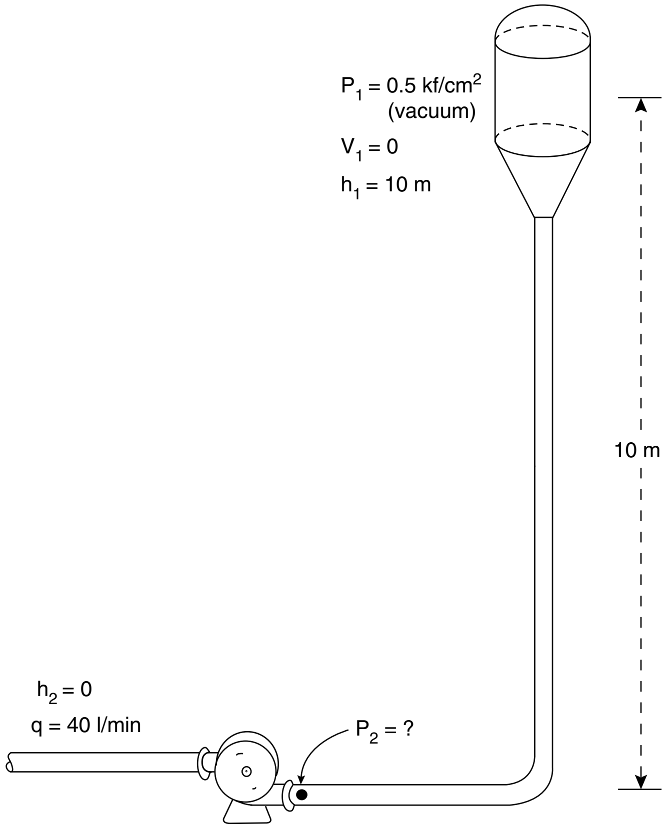


Figure 6.26 Diagram of piping system for catsup from a de-aerator to a pump.

$$q = 40\frac{\text{L}}{\text{min}} \cdot \frac{0.001\ \text{m}^3}{\text{L}} \cdot \frac{1\ \text{min}}{60\ \text{s}} = 0.0006666\ \text{m}^3/\text{s}$$

$$\text{Cross-sectional area, } A = \frac{\pi}{4}(0.06019)^2 = 0.002845\ \text{m}^2$$

From Table 6.2, D = 0.06019; R = 0.03009.

$$\bar{V} = \frac{q}{A} = \frac{0.0006666\ \text{m}^3/\text{s}}{0.002845\ \text{m}^2} = 0.2343\ \text{m/s}$$

The frictional resistance to flow can be calculated using Equation (6.45) after the Reynolds number has been calculated using Equation (6.50).

$$\text{Re} = \frac{8(0.2343)^{2-0.45}(0.03009)^{0.45}(1130)}{10.5\left[\dfrac{3(0.45)+1}{0.45}\right]^{0.45}} = 8.921$$

Flow is laminar and using Equation (6.45):

$$\frac{\Delta P_f}{L_\rho} = \frac{2(16/8.920)(0.2343)^2}{0.060019} = 3.272\,\mathrm{J/(kg\cdot m)}$$

where L = length of straight pipe + equivalent length of fitting.

From Table 6.3, the equivalent length of a 90-degree elbow is 35 pipe diameters.

$$\frac{\Delta P_f}{\rho} = 3.272(10.1) = 33.0\,\mathrm{J/kg}$$

$$L = 8 + 1(35)(0.06019) = 10.1\,\mathrm{m}$$

Substituting all terms in Equation (6.51):

$$P_2 = \left[\frac{52,00}{1130} + 98 - 33.0 - 0.03\right](1130)$$

$$\frac{52,000}{1130} + 9.8(10) + 0 + 0 = \frac{P_2}{1130} + 0 + \frac{(0.2343)^2}{2} + 33$$

$$= (46.02 + 9833.00.03)(1130)$$

$$= 125.4\,\mathrm{kPa\ absolute}$$

This problem illustrates one of the most important factors often overlooked in the design of a pumping system; that is, to provide sufficient suction head to allow fluid to flow into the suction side of the pump. As an exercise, the reader can repeat these same calculations using a pipe size of 1.0 in. at the given volumetric flow rate. The calculated absolute pressure at the intake side of the pump will be negative, a physical impossibility.

Another factor that must be considered when evaluating the suction side of pumps is the vapor pressure of the fluid being pumped. If the suction pressure necessary to induce the required rate of flow is lower than the vapor pressure of the fluid at the temperature it is being pumped, boiling will occur, vapor lock develops, and no fluid will flow into the intake side of the pump. For example, suppose the pressure at the pump intake is 69.9 kPa. From Appendix Table A.3, the vapor pressure of water at 90°C is 70 kPa. If the height of the fluid level is reduced to 4.8 m, the pressure at the pump intake will be 69 kPa, therefore, under these conditions, if the temperature of the catsup is 90°C and above, it would not be possible to draw the liquid into the intake side of the pump.

Example 6.22. The pump in Example 6.21 delivers the catsup to a heat exchanger and then to a filler. The discharge side of the pump consists of 1.5-in. stainless steel sanitary pipe 12 m long with two elbows. Joined to this pipe is the heat exchanger, which is 20 m of steam-jacketed 1-in. stainless steel sanitary pipe with two U-bends. (Assume the resistance of a U-bend is double that of a 90-degree elbow.) Joined to the heat exchanger is another 20 m of 1.5 in. pipe with two 90-degree elbows and the catsup is discharged into the filler bowl at atmospheric pressure. The pipe elevation at the point of discharge is 3 m from the level of the pump. The piping system before the pump and the rates of flow are the same as that in Example 6.21. Calculate the power requirement for the pump. The diagram of the system is shown in Fig. 6.27.

Solution:

From Example 6.21, $h_1 = 10$ m; $V_1 = 0$; $P_1 = 52$ kPa. $\Delta P_f/\rho$ for the section prior to the pump = 33 J/kg. $\Delta P_f/\rho$ for the section at the discharge side of the pump will be calculated in two steps: for the 1.5-in. pipe and the 1-in. pipe at the heat exchanger. D =1.5-in. pipe = 0.03561 m, R = 0.0178.

Equivalent length of four 90-degree elbows = 4(35)(0.03561) = 4.9854 m.

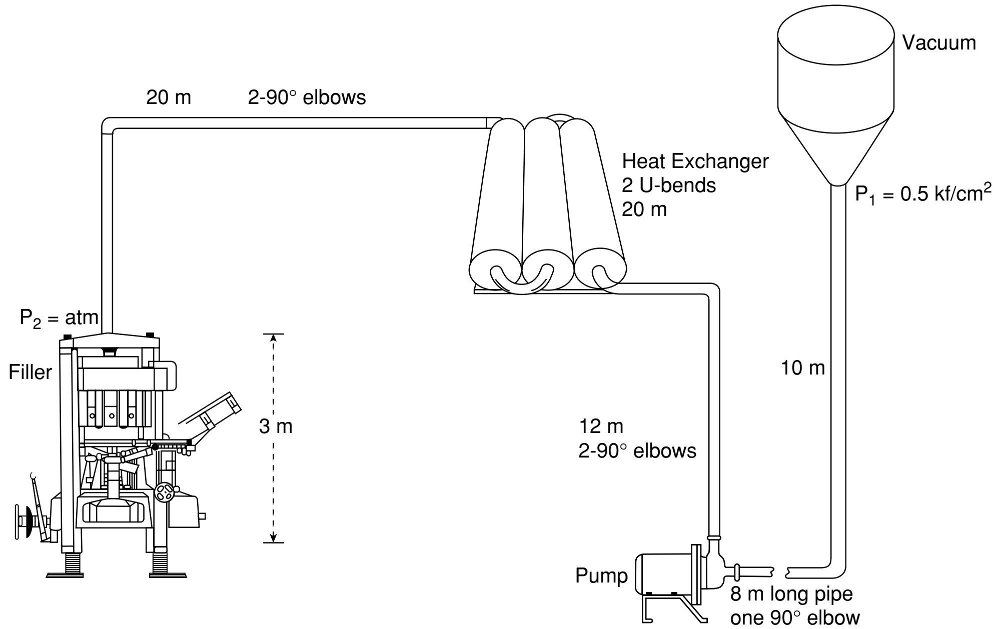


Figure 6.27 Diagram of a piping system for catsup from a de-aerator to a pump and heat exchanger and filler.

Total length = 20 + 12 + 4.99 = 36.99 m.

$$A = \frac{\pi}{4}(0.03561)^2 = 0.00099594\ m^2$$

$$\frac{\Delta P_f}{\rho} = \frac{2(16/35.82)(0.6693)^2(36.99)}{0.03561} = 415.7\ J/kg$$

$$\bar{V} = \frac{q}{A} = \frac{0.0006666\ m^3/s}{0.0009954\ m^2} = 0.6693\ m/s$$

$$Re = \frac{8(0.6693)^{2-0.45}(0.0178)^{0.45}(1130)}{10.5\left[\frac{3(0.45)+1}{0.45}\right]^{0.45}} = 35.87$$

D of 1-in. pipe = 0.02291; R = 0.01146 m.

$$A = \pi(0.01146)^2 = 0.0004126\,m^2$$

$$V = \frac{q}{A} = \frac{0.0006666\ m^3/s}{0.0004126\ m^2} = 1.616\ m/s$$

$$\frac{\Delta P_f}{\rho} = \frac{2(16/115.2)(1.616)^2(L)}{0.02291} = 31.64(L)\,J/kg$$

$$Re = \frac{8(1.616)^{2-0.45}(0.01146)^{0.45}(1130)}{10.5\left[\frac{3(0.45)+1}{0.45}\right]^{0.45}} = 115.3$$

The equivalent length of two U-bends = 2(2)(35)(D): L = 20 + 2(2)(35)(0.02291) = 23.2 m.

$$\frac{\Delta Pf}{\rho} = 31.64(23.27) = 734.0 \text{ J/kg}$$

Total resistance to flow = 33.0 + 415.7 + 734.0 = 1182.7 J/kg.
Substituting in Equation (6.51): P_2 = 101000 Pa; h_2 = 3 m; and V_2 = 0.6697 m/s.

$$\frac{52,000}{1130} + 9.8(10) + 0 + W_s = \frac{101,000}{1130} + 9.8(3) + \frac{(0.6697)^2}{2} + 1183.2$$

$$W_s = \frac{101,000}{1130} + 9.8(3) + \frac{(0.6697)^2}{2} + 1183.2 - \frac{52,000}{1130} - 9.8(10)$$

$$= 89.4 + 29.4 + 0.2 + 1182.7 - 46 - 98 = 1157.7 \text{ J/kg}$$

$$\text{Mass flow rate} = q(\rho) = 0.0006666(1130) = 0.7532 \text{ kg/s}$$

$$\text{Power} = (1157.7 \text{ J/kg})(0.7532 \text{ kg/s}) = 872 \text{ watts}$$

6.8 PUMPS

6.8.1 Types of Pumps and Their Characteristics

Pumps for transporting liquids may be classified into two general classes: *positive displacement* and *centrifugal*. Positive displacement pumps can operate effectively over a range of relatively slow drive shaft rotational speeds, and they deliver a fixed volume per revolution. These pumps can handle viscous liquids, generate high discharge pressure, are self-priming, and have some suction lift capability. Because of the relatively flat flow delivery within a moderate range of discharge pressure, positive displacement pumps are useful as metering pumps in processes where flow rate is a critical parameter for success of a process. Positive displacement pump flow must not be throttled otherwise high pressures will develop and break the pipes, destroy bearing seals in the pump drive shaft or burn up the drive motor because of the excessive load. In contrast, centrifugal pumps depend on an impeller rotating at high rotational speeds to increase the kinetic energy of fluid at the periphery of the impeller propelling the fluid at high velocity toward the pump discharge. Centrifugal pumps must be primed before they can generate any flow, or the source must be elevated above the pump to permit natural flooding of the pump inlet by gravity flow. Pressures generated are low and these pumps are not capable of pumping very viscous fluids. Fluid aeration, foaming, and fluid temperature elevation in the pump casing are other undesirable characteristics of centrifugal pumps particularly when operating at head pressures close to the maximum for the particular pump. However, when pressures against which the pump must work are low, and the fluid has relatively low viscosity, centrifugal pumps are the least expensive pump to use.

Positive displacement pumps may be classified as: reciprocating plunger or piston, gear; lobe, diaphragm, or progressing cavity pumps. Plunger or piston pumps generate the highest pressure but they deliver pulsating flow and develop pressure hammers, which might be detrimental to equipment downstream or to the piping system. Diaphragm pumps have the pumped fluid within a cavity formed by the diaphragm and the pump head cover. The diaphragm repeatedly moves forward and retracts. With each retraction of the diaphragm, the enlarged volume of the fluid cavity permits fluid to enter the cavity while forward motion of the diaphragm forces the fluid from the cavity toward the discharge. Check valves control the intake and discharge of fluid from the diaphragm cavity. A major advantage of diaphragm pumps is the complete isolation of the fluid from the surroundings while in the diaphragm cavity preventing contamination or aeration of the product. Diaphragm pumps are also available with an air drive rather than electric, and is a good feature in areas that are constantly wet. All the other positive displacement pumps have moving shafts that turn the pump rotor or gear, and these shafts must be adequately sealed to prevent fluid from leaking around the shaft. Progressing cavity pumps have the advantage of a gradual build-up of pressure as the fluid moves forward from the fluid intake at the opposite end of the pump from the discharge. Thus, the shaft seal for progressing cavity pumps only works against a fraction of the pressure generated at the pump discharge.

6.8.2 Factors to Be Considered in Pump Selection

Flow and *discharge pressure:* Pumps are sized according to the amount of fluid they deliver at designated discharge pressures. The latter is commonly called the *head*, which is defined as the height of a column of fluid that generates a pressure at its base equivalent to the pressure registered by a gauge placed at the discharge port of the pump. A graph of flow as a function of the head is called the *performance curve* of a pump and is available from pump manufacturers. Each make, model, and size of pump has a performance curve, which should be consulted to verify if a given pump can satisfactorily perform the required application.

Fluid viscosity: The viscosity of the fluid not only determines the head against which the pump must work but also the net positive suction head available at the suction inlet to the pump. For example, centrifugal pumps require a *net positive suction head* (NPSH) to be available, in the application under consideration, which is greater than that specified for a particular make and model of the pump. The NPSH is discussed im more detail later in this section. Generally, the limitation of centrifugal pumps in effectively pumping a fluid of high viscosity is the relatively high available NPSH they require compared to positive displacement pumps. However, even with positive displacement pumps, fluids with very high viscosity that would exhibit a very high pressure drop due to fluid friction in a short length of pipe connecting the suction to the fluid reservoir could prevent acceptable functioning of the pump for the desired application.

Fluid temperature affects the available NPSH. High fluid vapor pressure results in flashing of vapor to negate the suction developed by the pump and result in a condition called *vapor lock*. Fluid is prevented from entering the pump resulting in zero flow. To prevent vapor lock when pumping high temperature fluids, the fluid source must be elevated above the pump level in order that gravity flow will permit flooding of the pump inlet at the desired rate of flow.

Fragility of suspended particles: Fluids that contain fragile suspended particles require pumps that have large cavities to hold the particles without compressing them, and there should be a minimum of restrictions, sharp turns, and areas of low fluid velocities in the flow stream to prevent solids from bridging while flowing out of the pump.

Ease of cleaning: Fluids that are good substrates for microbiological growth will require a pump that can be easily cleaned. Pumps should be capable of being cleaned in place. Otherwise, the pump should be capable of being cleaned by simply removing the pump head cover without having to disconnect the pump from the connecting pipes.

Abrasiveness of suspended solids in the fluid: Suspended solids that are abrasive will require a pump where there is a minimum likelihood of having particles trapped between a stationary and moving part. Centrifugal and diaphragm pumps are ideal for this type of application.

Corrosiveness of fluid: Material of construction of any part of the pump that contacts an abrasive fluid must be of the type that will resist chemical attack of the fluid.

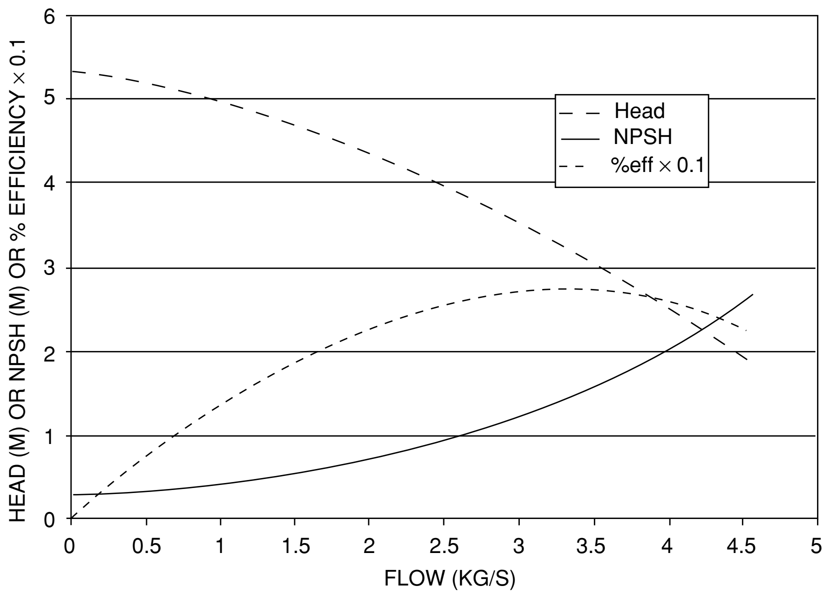


Figure 6.28 Performance curve of a volute type centrifugal pump.

6.8.3 Performance Curves of Pumps

The performance curve of a volute-type centrifugal pump is shown in Fig. 6.28. A characteristic of centrifugal pumps is the low head that they can work against. This particular pump was effectively shut off at a pressure equivalent to 5.3 m of fluid (51.9 kPa ga). Another characteristic is the required NPSH. For example, at the maximum pump efficiency, flow is 3.5 kg/s and the head is 2.8 m. This will require a NPSH of 1.5 m. The NPSH is defined as follows:

$$\text{NPSH available} = \left(\frac{1}{\rho g}\right)(P_{atm} \pm \rho gh - \Delta P_f - P_v)$$

where ρ, g, h, and ΔP_f are as previously defined. P_{atm} is the atmospheric pressure, and P_v is the fluid vapor pressure.

Example 6.23. A fluid with a density of 1004 kg/m^3 and a viscosity of 0.002 Pa s is to be pumped using the pump with a performance curve shown in Fig. 6.28. If the inlet pipe is 1.5 in. sanitary pipe, the suction pipe section is 1.2 m long with one 90-degree elbow, fluid source is 1 m below pump level, and the fluid vapor pressure is the same as that of water at 35°C (5.6238 kPa), calculate the NPSH available when operating at a head of 2.8 m, and determine if the pump will be suitable for operating under the given conditions. The atmospheric pressure is 101,000 Pa.

Solution:

The pipe diameter is 0.03561 m. The equivalent length of pipe $L = 1.2 + 1(35)(.03561) = 2.45$ m. At a head of 2.8 m, the flow rate is 3.5 kg/s.

$$\bar{V} = 3.5\ \frac{kg}{s} \cdot \frac{1m^3}{1004\,kg} \cdot \frac{1}{\pi\left(\frac{.03561}{2}\right)^2} = 3.5\frac{m}{s}$$

Using Poiseuille's equation:

$$\Delta P_f = \frac{8\ L\mu V}{R^2} = \frac{8(2.446)(0.002)(3.5)}{\left(\frac{0.03561}{2}\right)^2} = 432\ Pa$$

The NPSH available is greater than the 1.5 m required by the pump, therefore it will be possible to operate the pump under the specified conditions.

$$\text{NPSH available} = \frac{1}{(1004)(9.8)}\,[101000 - 9.8(1000)(1) - 432 - 5624] = 8.65\ m$$

The performance curve of a positive displacement pump shown in Fig. 6.29 depicts a flow rate that depends on the speed of the rotor. Because a fixed volume is delivered with each rotation of the rotor, the rate of flow is proportional to the rotational speed. Note that the rate of flow at 400 rpm is almost double that at 200 rpm. High discharge pressures generated are one order of magnitude greater than those generated by centrifugal pumps. The flow reduction with an increase in head is gradual. Flow reduction with increase in head depends on the rotor design. A tight clearance between the rotor and the casing minimizes backflow of fluid at high pressures thus maintaining a fairly flat flow rate with pressure. Positive displacement pumps generally develop more constancy of flow with increasing head as the viscosity of the fluid increases because backflow is reduced as fluid viscosity increases.

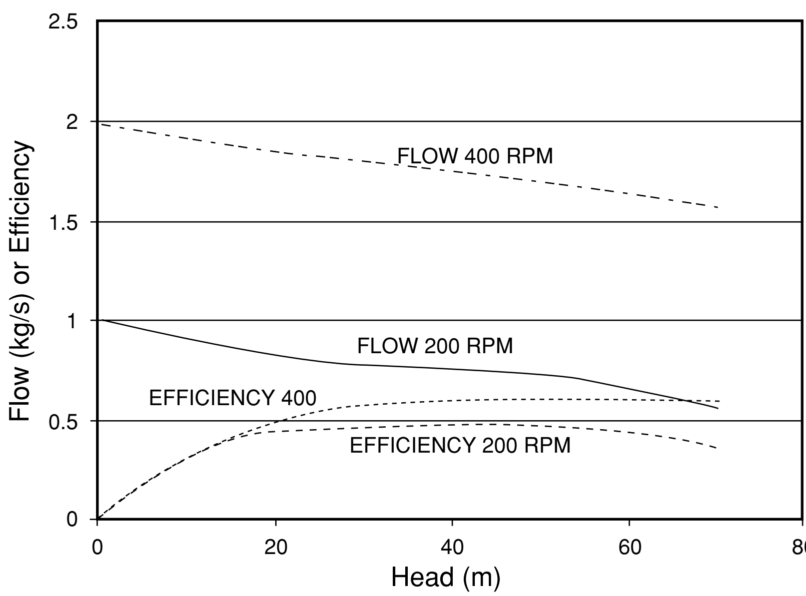


Figure 6.29 Performance curve for a two-lobe rotary positive displacement pump.

Attaining high pressures and maintaining constancy of flow with increasing pressures, however, is not desirable if these conditions are not required in an application. The reduction in backflow requires tight clearances between the rotor and casing, and to generate high pressures, there must be a tight shaft seal increasing friction thus reducing the pump efficiency.

The *pump efficiency* is defined as the ratio of the hydraulic horsepower to the brake horsepower. Let H = pressure at pump discharge expressed as height of fluid in meters, g = acceleration due to gravity in m/s^2 and $\bar{m}$ is the mass rate of flow in kg/s:

$$\text{Hydraulic Horsepower} = \frac{H \cdot g \cdot \bar{m}}{745.7}$$

The numerator is power in Watts and the denominator is 745.7 W/HP.
The brake horsepower is what the pump manufacturer specifies for the motor drive of a pump to deliver a specified rate of flow against a specified head. Efficiencies of centrifugal pumps and positive displacement pumps are rather low as shown in Figs. 6.28 and 6.29. Centrifugal pump efficiencies peak at 28% for the centrifugal pump shown in Fig 6.28, and the maximum efficiency occurs over a very narrow flow rate–head combination. In contrast, positive displacement pump efficiency (Fig. 6.29) remains relatively constant over a wide range of head. Higher rotor speeds induces higher efficiency than at lower speeds. Efficiencies for this particular pump were 50% or 60% at 200 and 400 rpm, respectively, at the optimum head and flow rate. Energy losses in pumps are attributable primarily to backflow and mechanical friction.

PROBLEMS

6.1. The following data were obtained when tomato catsup was passed through a tube having an inside diameter of 1.384 cm and a length of 1.22 m.

Flow rate (cm^3/s)	*P (dynes/cm^2)*
107.5	50.99×10^4
67.83	42.03×10^4
50.89	33.07×10^4
40.31	29.62×10^4
10.10	15.56×10^4
8.80	14.49×10^4
33.77	31.00×10^4
53.36	35.14×10^4
104.41	46.85×10^4

Determine the fluid consistency index K and the flow behavior index n of this fluid.

6.2. Figure 6.30 shows a water tower and the piping system for a small manufacturing plant. If water flows through the system at 40 L/min, what would be the pressure at point B? The pipes are wrought iron pipes. The water has a density of 998 kg/m^3 and a viscosity of 0.8 centipoise. The pipe is 1.5-in. (nominal) wrought iron pipe.

6.3. The catsup in Problem 1 (n = 0.45, K = 6.61 Pa · s^n) is to be heated in a shell and tube heat exchanger. The exchanger has a total of 20 tubes 7-m long arranged parallel inside a shell. Each tube is a 3/4-in. outside diameter, 18 gauge heat exchanger tube, (From a table

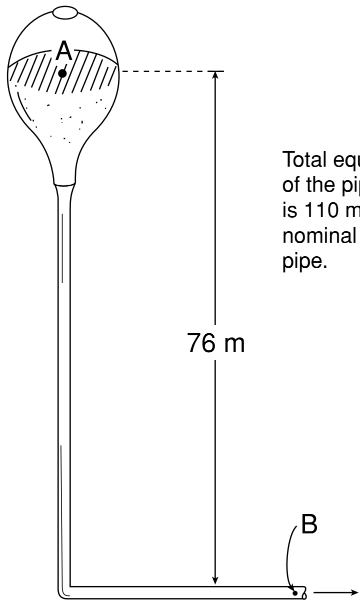


Figure 6.30 Water tower and piping system for Problem 2.

of thickness of sheet metal and tubes, an 18-gauge wall is 0.049 in.) It is possible to arrange the fluid flow pattern by the appropriate selection of heads for the shell, such that number of passes and the number of tubes per pass can be varied. Calculate the pressure drop across the heat exchanger (the pressure at the heat exchanger inlet necessary to push the product through the heat exchanger) if a flow rate of 40 L of product per min (density = 1013 kg/m^3) is going through the system for (a) two-pass (10 tubes/pass) and (b) five-pass system (four tubes per pass). Consider only the tube resistance and neglect the resistance at the heat exchanger heads.

6.4. Figure 6.31 shows a de-aerator operated at 381 mm Hg vacuum. (Atmospheric pressure is 762 mm Hg.) It is desired to allow a positive suction flow into the pump. If the fluid has a density of 1040 kg/m^3, the flow rate is 40 L/min, and the pipe is 1.5-in. sanitary pipe, calculate the height "h" the bottom of the de-aerator must be set above the pump level in order that the pressure at the pump intake is at least 5 kPa above atmospheric pressure. The fluid is Newtonian and has a viscosity of 100 centipoises.
[Note: Total length of straight pipe from first elbow below the pump to the entrance to the pump = 4 m. Distance from pump level to horizontal pipe = 0.5 m.]

6.5. Calculate the total equivalent length of 1-in. wrought iron pipe that would produce a pressure drop of 70 kPa due to fluid friction, for a fluid flowing at the rate of 50 L/min. The fluid has a density of 988 kg/m^3 and a viscosity of two centipoises.

6.6. Calculate the horsepower required to pump a fluid having a density of 1040 kg/m^3 at the rate of 40 L/min through the system shown in Fig. 6.32. $\Delta P_f/\rho$ calculated for the system is 120 J/kg. Atmospheric pressure is 101 kPa.

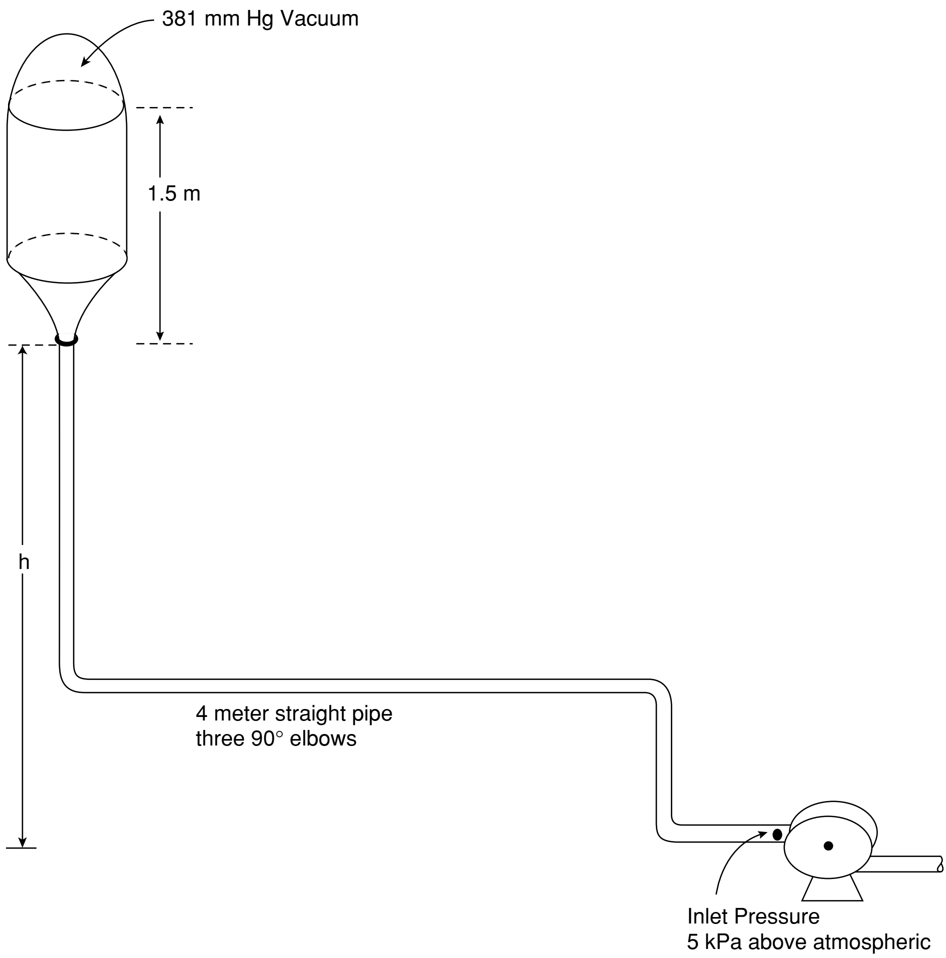


Figure 6.31 Diagram of Problem 4.

6.7. Calculate the average and maximum velocities of a fluid flowing at the rate of 20 L/min through a 1.5-in. sanitary pipe. The fluid has a density of 2030 kg/m^3 and a viscosity of 50 centipoises. Is the flow laminar or turbulent?

6.8. Determine the inside diameter of a tube that could be used in a high-temperature, short-time heater-sterilizer such that orange juice with a viscosity of 3.75 centipoises and a density of 1005 kg/m^3 would flow at a volumetric flow rate of 4 L/min and have a Reynolds number of 2000 while going through the tube.

6.9. Calculate the pressure generated at the discharge of a pump that delivers a pudding mix ($\rho = 995 \text{kg/m}^3$; $K = 1.0 \text{ Pa} \cdot s^n$, $n = 0.6$) at the rate of 50 L/min through 50 m of a 1.5-in. straight, level 1.5-in. stainless steel sanitary pipe. What would be the equivalent viscosity of a Newtonian fluid that would give the same pressure drop?

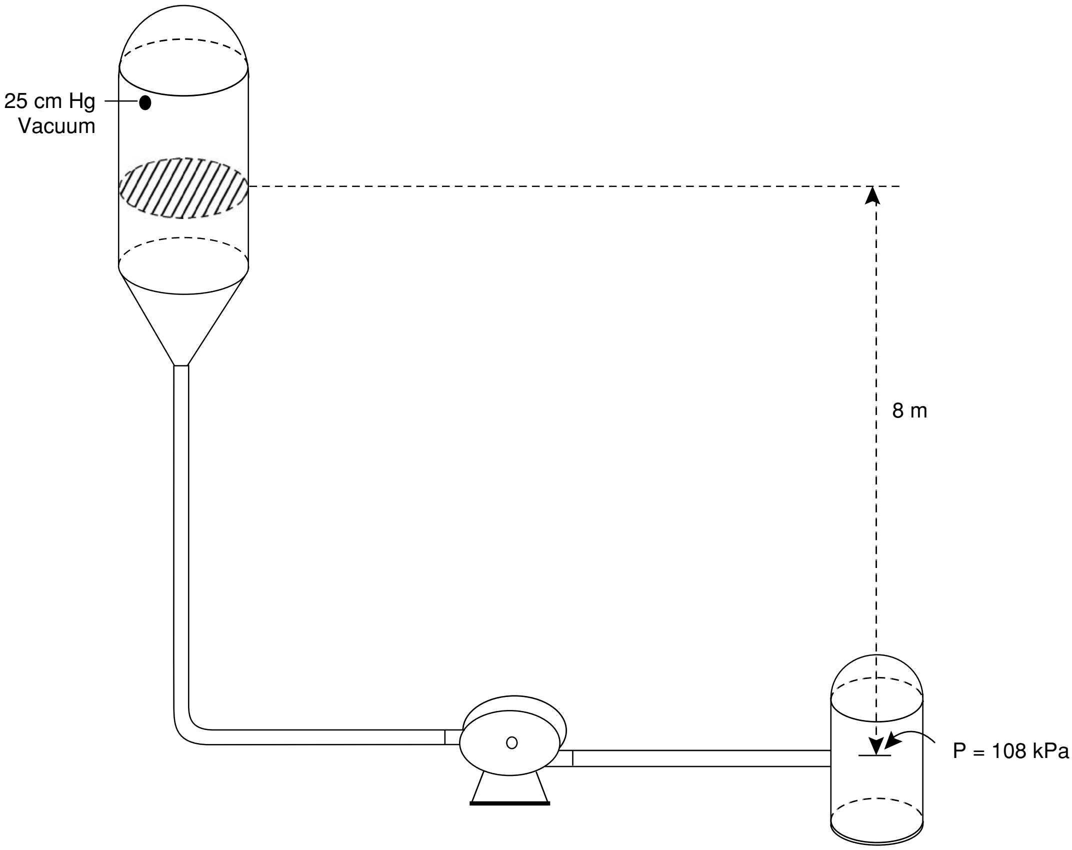


Figure 6.32 Diagram for Problem 6.

6.10. What pipe diameter will give a rate of flow of 4 ft/min for a fluid delivered at the rate of 2 gal/min?
6.11. Calculate the viscosity of a fluid that would allow a pressure drop of 35 kPa over a 5-m length of $^3/_4$-in. stainless steel sanitary pipe if the fluid is flowing at 2 L/min and has a density of 1010 kg/m^3. Assume laminar flow.
6.12. A fluid is evaluated for its viscosity using a Brookfield viscometer. Collected data of rotational speed in rev/min and corresponding apparent viscosity in centipoises, respectively, are 20, 7230; 10, 12060; 4, 25200; 2, 39500. Is the fluid Newtonian or non-Newtonian. Calculate the flow behavior index, n, for this fluid.
6.13. A fluid having a viscosity of 0.05 lb_{mass}/ft (s) requires 30 seconds to drain through a capillary viscometer. If this same viscometer is used to determine the viscosity of another fluid and it takes 20 seconds to drain, calculate the viscosity of this fluid. Assume the fluids have the same densities.
6.14. Figure 6.33 shows a storage tank for sugar syrup that has a viscosity of 15.2 centiposes at 25°C, and a density of 1008 kg/m^3. Friction loss includes an entrance loss to the drain pipe

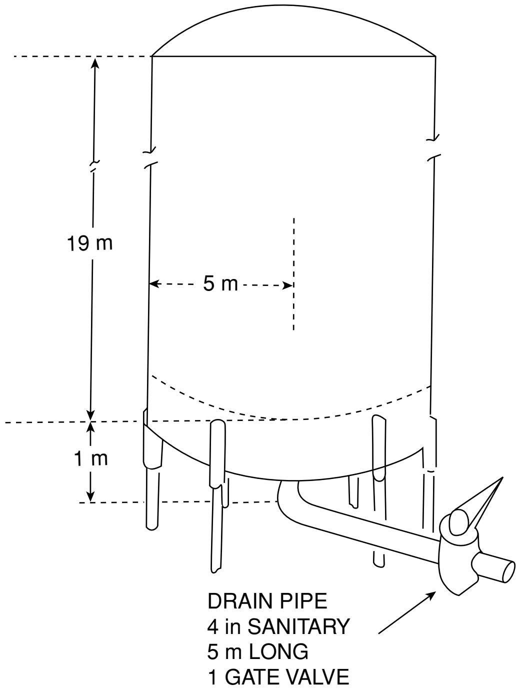


Figure 6.33 Diagram for Problem 14.

that equals the kinetic energy gain of the fluid, and resistance to flow through the short section of the drain pipe.

(a) Formulate an energy balance equation for the system.
(b) Formulate the continuity equation that represents the increment change in fluid level in the tank as a function of the fluid velocity through the drain pipe.
(c) Solve simultaneously equations formulated in (a) and (b) to calculate the time to drain the tank to a residual level 1 m from the bottom of the tank.
(d) Calculate the amount of residual fluid in the tank including the film adhering to the side of the tank.

6.15. A fluid tested on a tube viscometer 0.75 cm in diameter and 30 cm long exhibited a pressure drop of 1200 Pa when the flow rate was 50 cm^3/s.
(a) Calculate the apparent viscosity and the apparent rate of shear under this condition of flow.
(b) If the same fluid flowing at the rate of 100 cm^3/s through a viscometer tube 0.75 cm in diameter and 20 cm long exhibits a pressure drop of 1300 Pa, calculate the flow behavior and consistency indices. Assume wall effects are negligible.

6.16. Apparent viscosities in centipoises (cP) of 4000, 2500, 1250, and 850 were reported on a fluid at rotational speeds of 2, 4, 10, and 20 rev/min. This same fluid was reported to require a torque of 900 dyne cm to rotate a cylindrical spindle 1 cm in diameter and 5 cm high within the fluid at 20 rev/min.
(a) Calculate the flow behavior and consistency indices for this fluid.

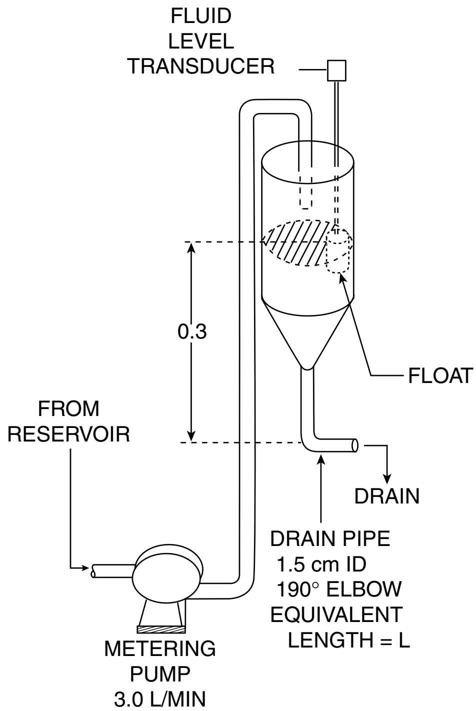


Figure 6.34 Diagram for Problem 19.

(b) When exhibiting an apparent viscosity of 4000 cP, what would have been the shear rate under which the measurement was made?

6.17. The flow behavior and consistency indices of whey at a solids content of 24% has been reported to be 0.94 and 4.46×10^{-3} Pa · s^n, respectively. If a rotational viscometer having a full scale torque of 673.7 dyne Acm is to be used for testing the flow behavior of this fluid, determine the diameter and height of a cylindrical spindle to be used such that at the slowest speed of 2 rev/min. The minimum torque will be 10% of the full scale reading of the instrument. Assume a length to diameter ratio of 3 for the spindle. Would this same spindle induce a torque within the range of the instrument at 20 rev/min?

6.18. Egg whites having a consistency index of 2.2 Pa · s^n and a flow behavior index of 0.62 must be pumped through a 1.5-in. sanitary pipe at a flow rate, which would induce a shear rate at the wall of 150/s. Calculate the rate of flow in L/min, and the pressure drop due to fluid flow resistance under these conditions.

6.19. The system shown in Fig. 6.34 is used to control the viscosity of a batter formulation used on a breading machine. A float indicates the fluid level in the reservoir, and by attaching the float to an appropriate transducer, addition of dry ingredients and water into the mixing tank may be regulated to maintain the batter consistency. The appropriate fluid level in the reservoir may be maintained when a different consistency of the batter is required, by changing the length of the pipe draining the reservoir. If the fluid is Newtonian with a viscosity of 100 cP, and

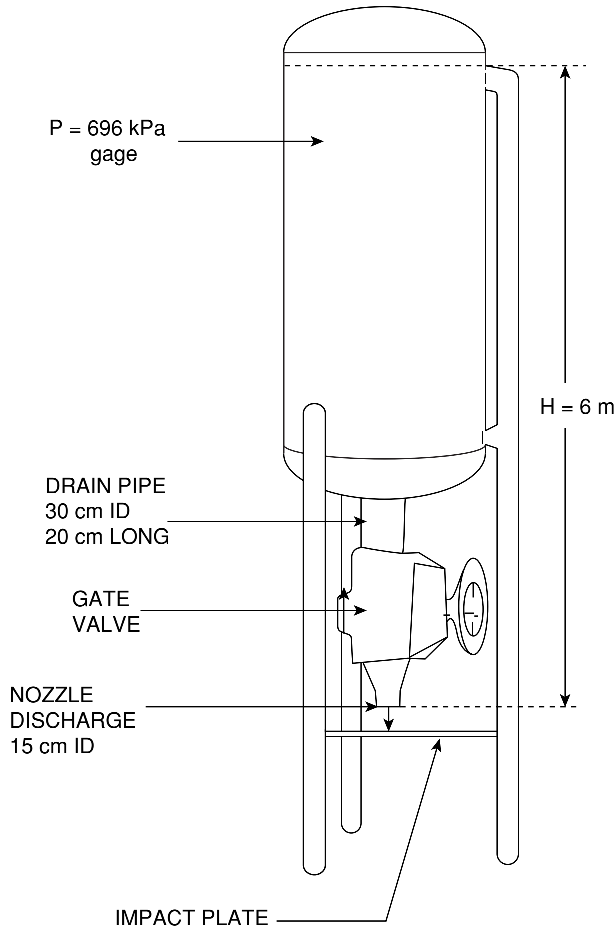


Figure 6.35 Diagram for Problem 20.

if fluid density is 1004 kg/m^3, calculate the feed rate that must be metered into the reservoir to maintain the level shown. Assume entrance loss is negligible compared to fluid resistance through the drain pipe.

6.20. The system shown in Fig. 6.35 has been reported to be used for disintegrating wood chips in the pulp industry, after digestion. It is desired to test the feasibility of using the same system on a starchy root crop such as cassava or sweet potatoes, to disrupt starch granules for easier hydrolysis with enzymes to produce sugars for alcoholic fermentation. In simulating the system on a small scale, two parameters are of importance: the shear rate of the slurry as it passes through the discharge pipe and the impact force of the fluid against the plate. Assume that entrance loss from the tank to the discharge pipe is negligible. Under the conditions shown, calculate the shear rate through the drain pipe, and the impact force against the plate at the time the drain pipe is first opened. The slurry has flow behavior and consistency indices of 0.7 and 0.8 Pa $\cdot$ s^n, and a density of 1042 kg/m^3.

6.21. In a falling film direct contact steam heater for sterilization, milk is pumped into a header that distributes the liquid to several vertical pipes, and the liquid flows as a film in laminar flow down the pipe. If the fluid has a density of 998 kg/m^3, and a viscosity of 1.5 cP, calculate the flow rate down the outside surface of each of 3.7-cm outside diameter pipes in order that the fluid film will flow at a Reynolds number of 500. Calculate the fluid film thickness when flow develops at this Reynolds number.

6.22. A sauce product is being formulated to match a reference product (Product A) that has a consistency index of 12 Pa s^n and a flow behavior index of 0.55. Rheological measurements of the formulated product (Product B) on a wide gap rotational viscometer using a cylindrical spindle 1 cm in diameter and 5 cm long are as follows, with speed in rev/min and torque in % of full scale, respectively: 2, 11; 4, 18; 10, 34; 20, 56. The viscometer constant is 7187 dyne cm. Calculate the apparent viscosity of Product A and Product B at 0.5 rev/min. At this rotational speed, did the apparent viscosity of Product B match that of Product A?

6.23. An FMC de-aerator 2.97 m high and 96.5 cm in diameter is rated to de-aerate from 4.2 to 8.4 kg/s of product. If the product has a density of 1008 kg/m^3, and has a flow behavior index of 0.44 and a consistency index of 8.1 Pa s ^{n}, calculate the film thickness and film velocity to achieve the mid-range (6.3 kg/s) of the specified capacity. If half the de-aerator height is to be covered by the fluid film, calculate the time available for any gas bubbles to leave the film into the vapor space in the chamber.

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